\( k \), \( X/k \) ab.var. \( g = \dim \nabla_X \)
\( \nabla_X \) dual,
\( \nabla_{\mathfrak{m}} \times \mathfrak{m} \times \mathfrak{m}^t \)

Fourier trans
\( \mathcal{F}: (\mathbf{CH}(X), \ast) \xrightarrow{\sim} (\mathbf{CH}(X^t), \cdot) \)

\( \mathcal{F}(\mathfrak{m}) = \mathfrak{m}_{X^t}(\mathfrak{m}^t_X(\mathfrak{m}) \cdot \mathfrak{m}(X)) \)

\( n \in \mathbb{Z}, \ [n]: X \to X \sim \)

\([n]^*, [n]_* : \mathbf{CH}(X) \to \mathbf{CH}(X) \)
proj. form: \( [n]_*[n]^* = n^{2g} \cdot \text{id} \)
\[ L \text{ line bun on } X \]

\[ L \text{ is symm. if } [-1] L = L \]
\[ \text{antisymm} \quad [-1] L = L^{-1} \]

If \( L \) symm then \( [n]^* L = L^n \)
\[ L \text{ antisymm} \quad [n] L = L_n \]

For any \( L \):
\[ L^2 = (L \otimes [-1] L) \otimes (L \otimes (-1) L^{-1}) \]
\[ \text{Symm} \quad \text{anti-symm} \]
\[ \text{Def: } i, j, s \in \mathbb{Z} : \]

\[ \text{CH}^i(X)_Q = \text{CH}^i_{(s)}(X) := \left\{ \alpha \in \text{CH}^i(X)_Q \mid \forall n : [n]^s \alpha = n^{2i-s} \cdot \alpha \right\} \]

\[ \text{CH}_j(X)_Q = \text{CH}^j_{(s)}(X) := \text{CH}^j_{(s)}(X) \]

\[ = \left\{ \alpha \in \text{CH}_j(X)_Q \mid \forall n : [n]_{2j+5} \alpha = n^{2j+5} \cdot \alpha \right\} \]
THEOREM (Beauville)

(i) \( CH_{(s)}^i (X) = \left\{ x \in CH^i (X) \mid F(x) \in CH^{0-i+s} (X^s) \right\} \)

and:

\( F: CH_{(s)}^i (X) \sim CH_{(s)}^{0-i+s} (X^s) \)

Similarly:

\( CH_{j, (s)}^j (X) = \left\{ x \in CH^j (X) \mid F(x) \in CH_{g-j-s} (X^s) \right\} \)

(ii) \( CH_{(s)}^i (X) \cdot CH_{(s)}^j (X) \subseteq CH_{(s+t)}^{i+j} (X) \)

\( CH_{i, (s)}^i (X) \star CH_{j, (s+t)}^j (X) \subseteq CH_{i+j, (s+t)} (X) \)

\( \sim \) bi-graded rings
(iii)

\[ \text{CH}^i(X) \oplus \bigoplus_{s = i-g} \text{CH}_{(s)}^i(X) \]

\[ \text{CH}_j(X) \oplus \bigoplus_{s = -j} \text{CH}_{j_1(s)}(X) \]
Represent the summand

$$\text{CH}_i^j(s)(X)$$

by a box in position \((2i - s, s)\), and call \(2i - s\) the weight. Example with \(g = 7\):
The usual grading by codimension of cycles is then represented by diagonal lines:
Fourier duality is now simply a reflection in the central vertical axis:
Conjecturally, all summands with $s < 0$ are zero; this is part of the Bloch–Beilinson Conjectures. As long as we do not know this, the picture would be as follows (example with $g = 9$):

conjecturally, all summands in this region vanish
In general, we only know the vanishing of the summands with $s < 0$ for the outer two layers:

![Diagram showing summands with $g = 9$ and all summands in the outer two layers vanishing.](image-url)
Let $H$ be any Weil cohom. $k$. for sm. proj. / $k$.

Exa:

- $k = \mathbb{C}$: Singular cohom. of $X(\mathbb{C})$.
- any $k$, prime $l \neq \text{char } (k)$: $l$-adic cohom.
- dR cohom.

Cycle class maps

$$\text{cl}: CH^i(X) \longrightarrow H^{2i}(X)$$

$X/k$ ab var. in any theory:

- $H^m(X) = \wedge^m H^1(X)$
- $[n]^* = \text{mult. by } n^m$ on $H^m(X)$
By weights:

\[ cl = 0 \quad \text{on all} \quad CH_{i(s)} \quad \text{with} \quad s \neq 0 \]

Conj. (?):

\[ cl \leftrightarrow \text{on} \quad CH_{i(0)} \]

If \( \alpha \in CH^i(X)_Q \) \( \text{w/} \) \( cl(\alpha) = 0 \)

then try Abel–Jacobi map

\[ \text{target space} \quad \text{"built out of"} \quad \]
\[ H^{2i-1} \]
If again:

\[ AJ(\alpha) = 0 \]

then (\ell\text{-adic coh}): go on using "higher AJ maps".

\[ \text{Filtration by } \stackrel{\leftarrow}{s} \]