

Lecture 4. ① $\mathbb{Z}_p^r \xrightarrow{\kappa} T(\mathbb{Z}_p)_t$

$r \leq g + p - 2$.

Question: how can we prove, without doing computations, that

$$\begin{array}{c} \uparrow \sim \\ J_b \\ U(\mathbb{Z}_p)_u \end{array}$$

$\kappa(\mathbb{Z}_p^r) \cap U(\mathbb{Z}_p)_u$ is finite? This should follow from our choice of L_1, \dots, L_{p-1} on J , namely that they are \mathbb{Z} -lin. ind. in $NS(J_{\mathbb{Q}})$.

② $A_{\mathbb{F}_p}^r \xrightarrow{\bar{\kappa}} T_{T_{\mathbb{F}_p}}(t)$ can be non-injective,

for example if $p=1$, $J=T$, and

$$J(\mathbb{Z})_{\circ_{\mathbb{F}_p}} = \ker(J(\mathbb{Z}) \rightarrow J(\mathbb{Z}/p^2\mathbb{Z})). \quad (\text{then } \bar{\kappa} = 0).$$

Two aims. (1) Show that \bar{A} as in Thm 4.12 can be computed, i.e. make it explicit.

(2) Show that the computations are not so bad.

(1): § 6-7.

§ 6. "Rigidify line bundles on C at b ".

$$\text{Pic}_{C/S}(T) = \frac{\text{Pic}(C_T)}{\text{Pic}(T)} =$$

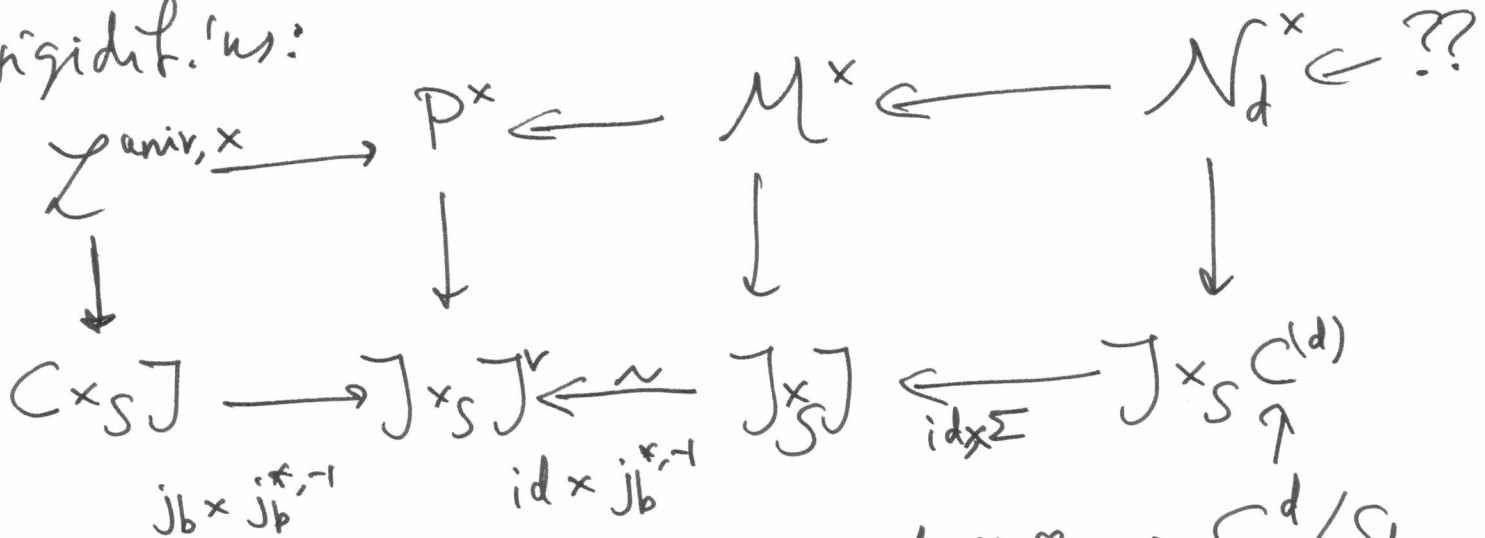
$$\begin{array}{c} C \\ \downarrow \\ T \rightarrow S \end{array} \quad \mathbb{Z}[1/n]$$

$$\left\{ (L, \varphi) : \begin{array}{l} L \text{ inv. } \mathcal{O}\text{-module on } C_T \\ \varphi: \mathcal{O}_T \xrightarrow{\sim} b_T^* L \end{array} \right\} \Big/ \cong$$

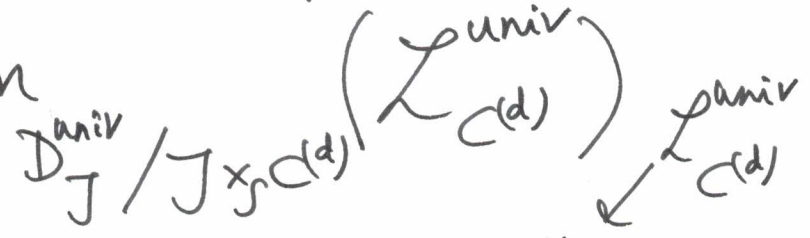
such objects have no nontrivial automorphisms.

Prop. 6.3.2 S any scheme, $C \rightarrow S$ proper smooth curve, of genus $g \geq 1$, $b \in C(S)$, $d \in \mathbb{Z}_{\geq 0}$

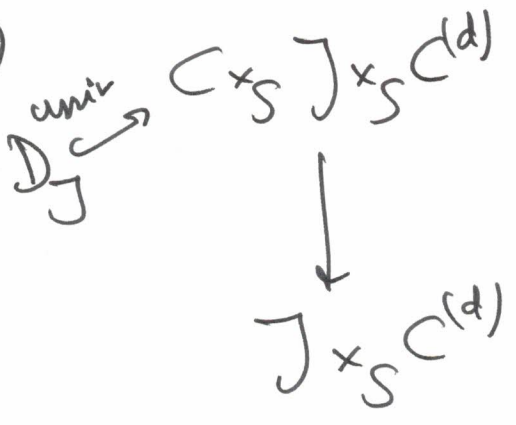
$\exists!$ morphisms of \mathbb{G}_m -torsors, compatible with rigidifications:



with: $N_d := \text{Norm}$



where: $\mathcal{D}_J^{univ} \rightarrow C \times C^{(d)}$
 finite loc. free of degree d .

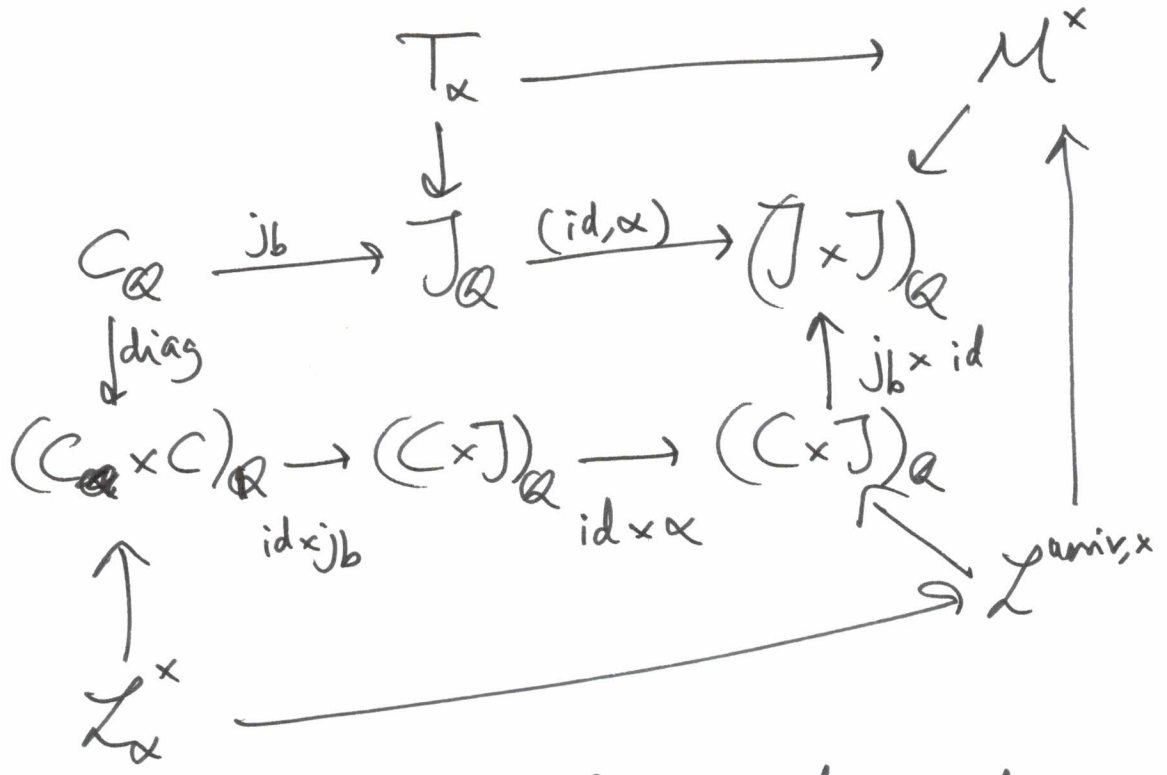


For example (6.3.12)

$$x_i, y_i, u_j, v_j \in \mathbb{C}(S)$$

$$\begin{aligned} \mathcal{M} \left(\sum_i (x_i - y_i), \sum_j (u_j - v_j) \right) &= \\ &= \bigotimes_j \left(u_j^* \mathcal{O}_{\mathbb{C}} \left(\sum_i (x_i - y_i) \right) \otimes_{\mathcal{O}_S} v_j^* \mathcal{O}_{\mathbb{C}} \left(\sum_i (y_i - x_i) \right) \right) \end{aligned}$$

§ 7. $\alpha := \text{tr}_{b_i} \circ f_i : J_{\mathbb{Q}} \rightarrow J_{\mathbb{Q}}$; make explicit.



$j_b^* T_{\alpha}$ trivial on $C_{\mathbb{Q}}$ \iff L_{α} trivial on diagonal in $(C \times C)_{\mathbb{Q}}$.

Computations in $T(\mathbb{Z}/p^2\mathbb{Z}), J(\mathbb{Z}/p^2\mathbb{Z}), J(\mathbb{F}_p)$. ^{4.}

1. $J(\mathbb{Z}/p^2\mathbb{Z}), J(\mathbb{F}_p)$: line bundles up to isom. on $C_{\mathbb{Z}/p^2\mathbb{Z}}$ or $C_{\mathbb{F}_p}$. , e.g. work with divisors.

2. $T(\mathbb{Z}/p^2\mathbb{Z})$: no extra effort if you use rigidified line bundles.

Finally we get back to J , good or bad?

Compare with $Gr_{2g, g}$ Grassmannian of g -dim. subsp. in a $2g$ dim. space.

dim: $\begin{array}{|c|c|} \hline a & a \\ \hline a & a \\ \hline \end{array} / \begin{array}{|c|c|} \hline a & a \\ \hline a & a \\ \hline \end{array} \quad g^2.$

embedded in: $\mathbb{P}^{\binom{2g}{g}-1} \quad \binom{2g}{g} \geq \frac{2^{2g}}{2g+1}.$

Not a problem: just describe subspaces by giving a basis.

Now J. Just the same.

Fix \mathcal{L} on C of degree $\geq 3g-1$.

Then $\forall D \geq 0$ eff. div. degree g

$$H^0(C, \mathcal{L}(-D)) \hookrightarrow H^0(C, \mathcal{L})$$

is a codim. g subspace.

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