$$
\begin{tikzcd}
C(\mathbb{Z}) \arrow[r] \arrow[d] & T(\mathbb{Z}) \\
 & \overline{T(\mathbb{Z})} \arrow[u, phantom, "\cap"] \\
C(\mathbb{Z}_p) \arrow[r] & T(\mathbb{Z}_p) \arrow[u, phantom, "\cap"]
\end{tikzcd}
$$

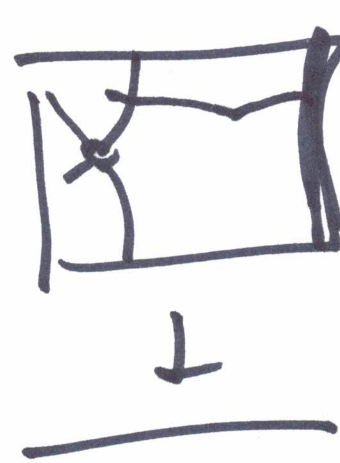
came from

problem with primes q s.t. $C^{sm}_{\mathbb{F}_q}$ reducible

$$
\begin{tikzcd}
 & & T \arrow[r] \arrow[d] \arrow[dr, phantom, "\ulcorner"] & P^{\times,\rho-1} \arrow[d] \\
U \arrow[r, hook] \arrow[urr, bend left] & C^{sm} \arrow[r] \arrow[ur, dashed] & J \arrow[r] & J \times J^{v,0,\rho-1}
\end{tikzcd}
$$

open subsch. of C where $C \to \operatorname{Spec} \mathbb{Z}$ is smooth

open subsets of C^{sm} obtained by removing in all fibres over closed pts of $\operatorname{Spec} \mathbb{Z}$ all irred. comp. except 1 that is geom. irred.



C^{sm}

$\operatorname{Spec} \mathbb{Z}$

For $P \in C(\mathbb{Z}) = C^{sm}(\mathbb{Z})$, P selects 1 geom. irr. comp. in each fibre

Say $n =$ prod. of primes q of bad red. of $C/\mathbb{Z}$. Then $C \longrightarrow \operatorname{Spec} \mathbb{Z}$ is smooth over $\mathbb{Z}[1/n]$.

There are finitely many of such U's, and $C_{\mathbb{Q}}(\mathbb{Q}) = C^{sm}(\mathbb{Z}) = \coprod_{\text{all } U\text{'s}} U(\mathbb{Z})$.

<u>Ex. in §8.</u> $y^2 + y = x^6 + \cdots$

$n = 3 \cdot 43$ $\quad C^{sm}_{\mathbb{F}_{43}}$ irred. $\quad C^{sm}_{\mathbb{F}_3}$:

$\exists$ exactly 2 U's.

$$J^{\vee,0} \hookrightarrow J^{\vee} \twoheadrightarrow \Phi^{\vee} \leftarrow \text{gr. sch. of components of } J^{\vee}$$

$\Phi^{\vee}$ trivial over $\mathbb{Z}[1/n]$

finite etale fibres over $\mathbb{Z}/n\mathbb{Z}$

$m :=$ l.c.m. of the ~~orders~~ exponents of $\Phi^{\vee}(\mathbb{F}_q)$, $q | n$

$$T(\mathbb{Z}) \hookrightarrow \overline{T(\mathbb{Z})} \subset T(\mathbb{Z}_p)$$

$\uparrow$ how to describe this?

$p \nmid n,\ p > 2.$

$$t \in T(\mathbb{F}_p) \leftarrow T(\mathbb{Z}), \quad t \mapsfrom \tilde{t}$$

$$\begin{tikzcd}
 & \overline{T} \arrow[r] \arrow[d, "\Gamma"] & P^{\times, \mu-1} \arrow[d] \\
U \arrow[ur, "\tilde{j}_b"] \arrow[r, "j_b"] & J \arrow[r, "{(id, (m_i \circ tr_{b_i} \circ f_i)_i)}"'] & J \times J^{r, g, \mu-1}
\end{tikzcd}$$

$u \in U(\mathbb{F}_p)$

$$x_{\tilde{t}} \in J(\mathbb{Z})_{j_b(u)} \to j_b(u) \in J(\mathbb{F}_p)$$

not from geom

$$J(\mathbb{Z}) \to T(\mathbb{Z})$$

$$J(\mathbb{Z})_0 \cong \mathbb{Z}^r$$

$x_1, \dots, x_r$

We get a map

$$\kappa_{\mathbb{Z}} : \mathbb{Z}^r \longrightarrow T(\mathbb{Z})_t$$

not really surjective.

Thm 4.10.

evaluate $\frac{1}{p}$ · parameters of $\Theta_{T,t}$

$$\begin{tikzcd}
\mathbb{Z}^r \arrow[r, "\kappa_{\mathbb{Z}}"] \arrow[d, phantom, "\cap \text{ dense}"] & T(\mathbb{Z}_p)_t \arrow[r, "\sim"] & \mathbb{Z}_p^{g+p-1} \\
\mathbb{Z}_p^r \arrow[urr] & &
\end{tikzcd}$$

$\exists!\ \kappa = (\kappa_1, \ldots, \kappa_{g+p-1})$

$$\kappa_i \in \mathbb{Z}_p\langle z_1, \ldots, z_r\rangle = \mathbb{Z}_p[z_1, \ldots, z_r]^{\wedge p}$$

and $\overline{T(\mathbb{Z})_t}$ = image of κ.

Proof: all of §5, 3.5 pages log & exp.

$n \mapsto nP = \exp(n \cdot \log P)$

$$\begin{tikzcd}
 & \mathbb{Z}_p^r \arrow[d, "\kappa"'] \arrow[ddr, bend left, "{\kappa=(\kappa_1,\dots,\kappa_{g+p-1})}"] & \\
U(\mathbb{Z})_u \arrow[d] & \overline{T(\mathbb{Z})_t} \arrow[d, phantom, "\cap"] & \\
U(\mathbb{Z}_p)_u \arrow[r, hook, "\sim"'] & T(\mathbb{Z}_p)_t \arrow[r, "\sim"] & \mathbb{Z}_p^{g+p-1}
\end{tikzcd}$$

$\Downarrow$ comes at p from a closed imm.

1. we want to pull back equations for the complete int. $U \hookrightarrow T$ at u get $g+p-2$ equations. to $\mathbb{Z}_p^r$.

2. We want to do this in terms of formal geometry, rings like $\mathbb{Z}_p\langle z_1, \dots, z_r\rangle$, and then reduce mod p, get polynomials in $\mathbb{F}_p[z_1, \dots, z_r]$