

Geometric Quadratic Chabauty, $\frac{0}{1}$.

(jt. work with Guido Lido).

C nice curve / \mathbb{Q} , assume $b \in \mathbb{Q}$.

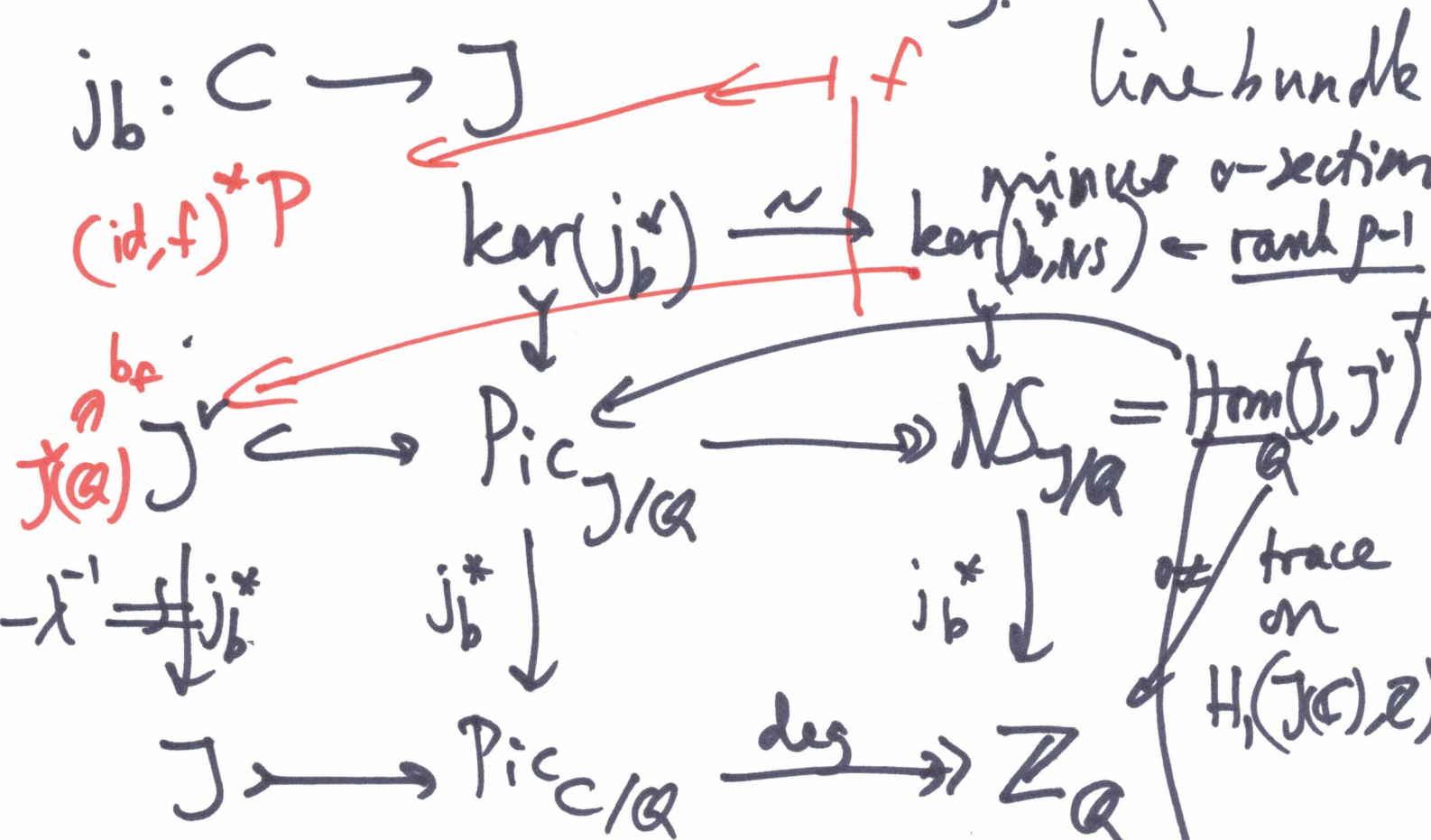
$$j_b: C \xrightarrow{571.} J, P \mapsto \mathcal{O}_C(P-b).$$

Chabauty has problem if $r \neq g$.

$$\begin{array}{ccc} \mathbb{Q} & \longrightarrow & J(\mathbb{Q}) \\ \downarrow & & \overline{J(\mathbb{Q})} \leftarrow \text{can be equal.} \\ C(\mathbb{Q}_p) & \longrightarrow & J(\mathbb{Q}_p) \end{array}$$

Idea: replace J by something bigger, higher dimension, and then play Chab game.

What to take? A G_m -torsor on J . !



Given $f: J \rightarrow J^v$ with trace \circ ,
 get a line bundle on J
 that is trivial on $j_b(C)$.

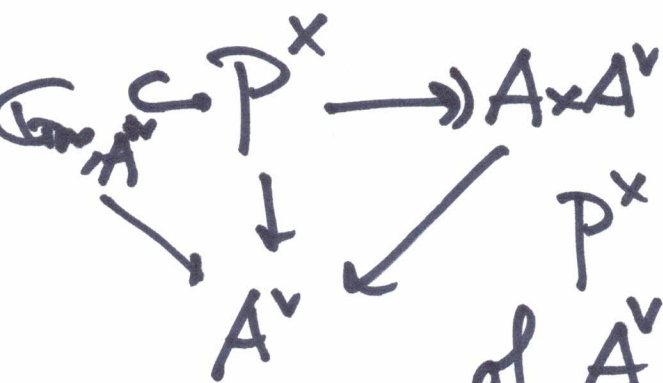
Poincaré bundle.

2.

\forall ab. var. A , can view A^\vee as $\text{Ext}^1(A, \mathcal{G}_m)$. $\mathcal{G}_m \hookrightarrow E \twoheadrightarrow A$

So over A^\vee have univ. extension:

are rigid. no non-trivial autom. that induce id_A and $\text{id}_{\mathcal{G}_m}$.



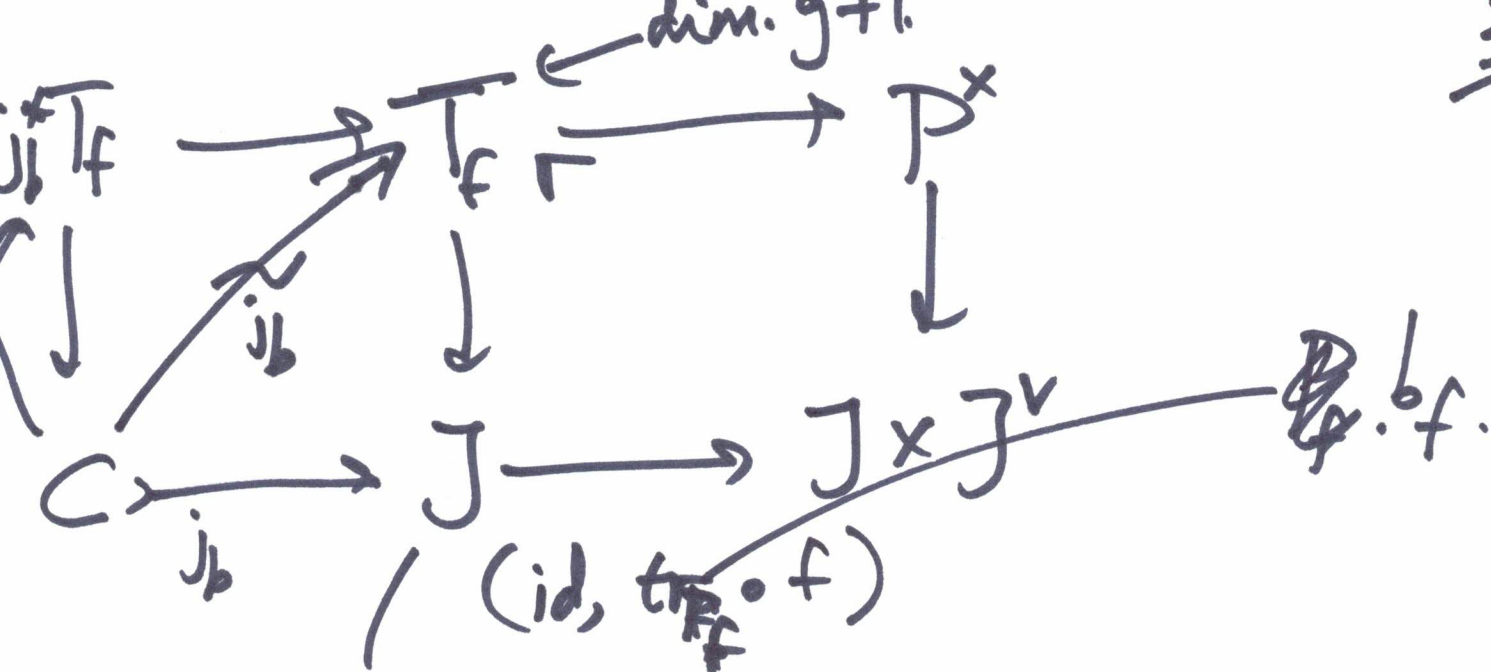
P^x is also the univ. ext. of A^\vee by \mathcal{G}_m , over A .

P^x is a \mathcal{G}_m -biextension of $A \times A^\vee$:
2 partial gr. laws: $z_1 + z_2 \in P^x(x_1 + x_2, \gamma)$

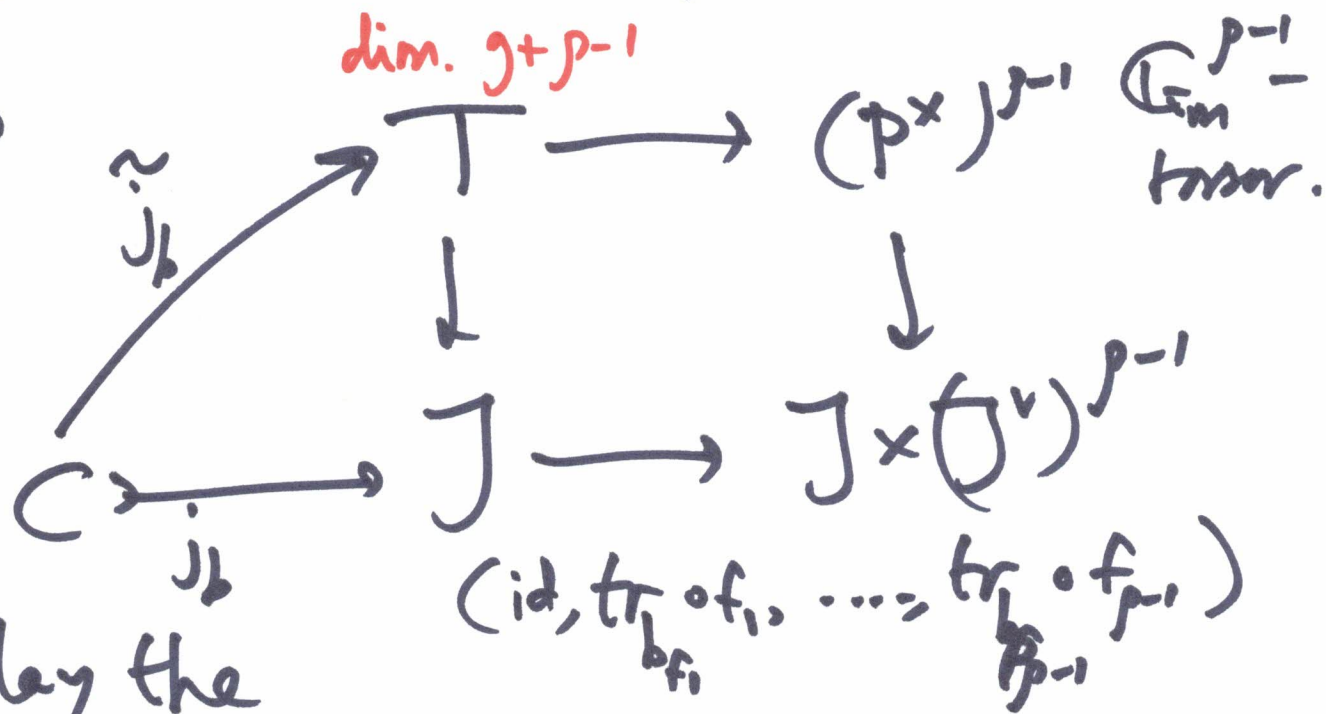
$x_1, x_2 \in A, \gamma \in A^\vee$

$(x_1, \gamma) \in A \times A^\vee \quad z_1 \in P^x(x_1, \gamma)$

$(x_2, \gamma) \in A \times A^\vee \quad z_2 \in P^x(x_2, \gamma)$



Take a \mathbb{C} -basis f_1, \dots, f_{p-1} of $\ker(j_b^+ NS)$,
 gives



We play the Chabauty game in T .
 Hope that it works if $r < g+p-1$.
 (most wanted ex. have $p=g$.)

$T(\mathbb{Q})$ is a $\mathbb{Q}^{\times}, \mathcal{P}^{-1}$ -torsor.



$\mathbb{Q}^{\times} = \{\pm 1\} \times \mathbb{Z}$ (set of primes)

$J(\mathbb{Q})$ Big problem, too many \mathbb{Q} -points in T .

Solution: extend the geometry over \mathbb{Z} . $\mathbb{Z}^{\times} = \{\pm 1\}$.

From now on everything over \mathbb{Z} .

C proper regular model of $C_{\mathbb{Q}}$.

$J :=$ Néron model of $J_{\mathbb{Q}}$.

$J^{\vee} :=$ ————— $J^{\vee}_{\mathbb{Q}}, J^{\vee,0} \subset J^{\vee}$:

Fiberwise conn. comp. of ν .
 P^{\times} : unique extension of $P_{\mathbb{Q}}$ to $J \times J^{\vee,0}$ as bis-extension.

We use the biext. thr. of P^x 5.
 over $J \times J^{v,0}$ to parametrize.

