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ht 2

Γ over k

$$\ell(x) = \sum \frac{x^{p^{2n}}}{p^n}$$

σ universal deformation

$$W[[u_1]] = E_0$$

$$f(x) = x + \frac{u_1}{p} f^{\varphi}(x^p) + \frac{1}{p} f^{\varphi^2}(x^{p^2})$$

$$f(x) = \sum m_n x p^n$$

$$m_n = \frac{u_1}{p} m_{n-1}^{\varphi} + \frac{1}{p} m_{n-2}^{\varphi}$$

$$w = \lim_{n \to \infty} p^n m_{2n}$$

$$u w_1 = \lim_{n \to \infty} p^n m_{2n-1}$$

$$A = \begin{bmatrix} \frac{u_1}{p} & \frac{1}{p} \\ 1 & 0 \end{bmatrix} = A(u_1)$$

$$\begin{pmatrix} m_n & \frac{1}{p} m_{n-1}^{\varphi} \\ m_{n-1} & \frac{1}{p} m_{n-2}^{\varphi} \end{pmatrix}^{\varphi} A = \begin{pmatrix} m_{n+1} & \frac{1}{p} m_n^{\varphi} \\ m_n & \frac{1}{p} m_{n-1}^{\varphi} \end{pmatrix}$$

$$\begin{pmatrix} \omega & \frac{1}{p}(\omega\omega_1)^{\varphi} \\ \omega W_1 & \omega^{\varphi} \end{pmatrix}$$

$$= \lim_{n\to\infty} A^{\varphi^n} \cdots A\, A(0)^{-(n+1)}$$

$\underline{X}$ Riemann Surface

$$\begin{tikzcd} H^0(\underline{X}; \Omega^1) \arrow[r, "\text{Hodge}"] & H^1_{DR}(\underline{X}) \\ & H^1(\underline{X}; \mathbb{Z}) \arrow[u] \end{tikzcd}$$

de Rham coh.

S

V = Lie variety over S of dim n

$\mathbb{A}^n$ $\qquad$ $\mathrm{Map}(V, \mathbb{A}^1)$

$= S[[x_1 \ldots x_n]]$

$$\Omega^0 V \xrightarrow{d} \Omega^1 V \cdots \xrightarrow{d} \Omega^n V$$

$H^*_{DR}(V)$

$S = W$ $\qquad$ $\dim V = 1$ $\qquad$ $= \mathbb{A}^1$

$d\left(\frac{x^{mp^k}}{p^k}\right)$

$$S[[x]] \xrightarrow{d} S[[x]]\,dx$$

$H^0_{DR}(V) = S$

$$H^1_{DR} = \prod_{\substack{(m,p)=1 \\ k \geq 1}} S/p^k$$

Functorial in S

V

- $S \xrightarrow{r} T$

$$T \otimes_S H^*_{DR}(V) \xrightarrow{r^*} H^{\cdot}_{DR}(\mathcal{B}\, r^*V)$$

- $S \underset{g}{\overset{f}{\rightrightarrows}} T$ $\qquad I \subset T$

ideal
with divided
powers

$f \equiv g \mod I$

$$\boxed{\text{Then } f^* = g^*}$$

G = formal gp

$\cancel{V \times V \xrightarrow{\mu} V}$

$$\begin{tikzcd} & G \\ G \times G \arrow[ur] \arrow[r, "\mu"] \arrow[dr] & G \\ & G \end{tikzcd}$$

$\mu^* \quad H_{DR}(G) \longrightarrow H_{DR}(G \times G)$

$\omega \in H^*_{DR}(G)$ is <u>primitive</u>

if $\quad \mu^* \omega = \pi_1^* \omega + \pi_2^* \omega$

$D(G)$ = prim in

$H^1_{DR}(G)$

$G = G_a \qquad \dim G = 1$

$\cancel{d(x^{p^m}} \qquad d\left(\frac{x^{np^k}}{p^k}\right)$

$$\frac{x^p}{p} \longrightarrow \underbrace{\frac{(x+y)^p}{p} - \frac{x^p}{p} - \frac{y^p}{p}}_{\text{coeff in } W((x))}$$

$d \downarrow$ | $d = 0$ in H^1_{DR}

$x^{p-1}dx$

so $d\left(\frac{x^p}{p}\right) \in D(G)$

$d\left(\frac{x^{2p}}{p}\right) \qquad d\left(\frac{x^{p^k}}{p}\right) \in D(G)$

$D(a)$ $\quad \prod_k \mathbb{Z}/p$

$d\left(\frac{x^{p^k}}{p}\right)$

$G \qquad \ell(x) = \sum \frac{x^{p^n}}{p^n}$

$\mu^* \ell(x) = \ell(x) + \ell(y)$

$\ell(x \underset{a}{+} y)$

$d\ell(x) \in D(a)$

$d\,\frac{1}{p}\,\ell(x^p) \in D(a)$

$$\frac{1}{p}\ell(x^{p^2}) = \ell(x) - x$$

$$\frac{1}{p}\ell(x^{p^k})$$

$D(a)$ has basis

$$d\,\ell(\cancel{x}) \qquad d\,\frac{1}{p}\ell(x^p)$$

G^{univ}

$$f(x) = x + \frac{u_1}{p} f^{\varphi}(x^p) + \frac{1}{p} f^{\varphi^2}(x^{p^2})$$

$D(G^{univ})$ basis

$$\begin{cases} d\,f(x) \\ d\,\frac{1}{p} f^{\varphi}(x^p) \end{cases}$$

Γ $\quad k \quad \hat{A} \quad \mathbb{W}$

$\leftarrow \log \ell(x)$

$G^{univ} \quad \mathbb{W}[[u_i]]$

$G_0 \quad \mathbb{W}/p[[u_i]] = k[[u_i]]$

$$H^{0,1}(G_{univ}) \to D(G_{univ}) \leftarrow \text{depends only on } G_0$$

$\uparrow$ depends on G_{univ}.

$\uparrow$?

$$H^0(X;\Omega^1) \to H^1_{DR}(X)$$

$\uparrow$

$H^1(X;\mathcal{O})$

$$S \quad\quad G$$

$$\downarrow$$

$$S/pS \quad\quad G_0$$

$D(G)$ depends only on G_0

(p) has divided powers

$\tilde{G}$ a different lift

$$A' \times A' \underset{\cancel{\tilde{G}}\ \tilde{\hat{\mu}}}{\overset{\cancel{G}\ \mu}{\rightrightarrows}} A'$$

$$\mu^* = \hat{\mu}^* : H^1_{DR}(A') \to H^1_{DR}(A' \times A')$$

Frobenius $\quad G_0 \quad\quad x \to x^p$

$$\cancel{\varphi^{\circ} G_0 \to G}$$

$$G_0 \to \varphi^{\circ} G_0$$

$$\Rightarrow \varphi^{\circ} D(G) \xrightarrow{F} D(G)$$

$$\frac{1}{p} f^{\varphi^k}(x^{p^k}) \to \frac{1}{p} f^{\varphi^{k+1}}(x^{p^{k+1}})$$

$$f(x) = x + \frac{u_1}{p} f^{\varphi}(x^p) + \frac{1}{p} f^{\varphi^2}(x^{p^2})$$

$$df \quad d\left(\frac{1}{p} f^{\varphi}(x^p)\right)$$

$$F \quad \begin{pmatrix} 0 & 1 \\ p & -u_1 \end{pmatrix}$$

$$\begin{tikzcd}
\mathbb{W}[[u_1]] \arrow[rr, shift left, "r"] \arrow[rr, shift right, "r_0"'] \arrow[dr] & & \mathbb{W}\langle\langle u_1\rangle\rangle \\
& \mathbb{W} \arrow[ur] &
\end{tikzcd}$$

$u_1 \mapsto 0$

$\widetilde{\Gamma}$

$$\begin{array}{ccc}
M & = & D(G^{\cancel{univ}}_{u_1=0}) \\
B \downarrow & & d\ell(x) \\
 & & d\frac{\ell(x^p)}{p}
\end{array}$$

$H^{01}(G^{univ}) \longrightarrow r^{*} D(G^{univ})$

$$\begin{tikzcd}
H^0(\Omega^1) \arrow[r] & H^1_{DR}(X) \\
& H^1(X;\mathbb{Q}) \arrow[u]
\end{tikzcd}$$

$\widetilde{\Gamma}$ has log $\ell(x) = \sum \frac{x^{p^{2n}}}{p^n}$

period

$$\begin{tikzcd} & M \\ H^{01}(G^{univ}) \arrow[r] & D(G^{univ}) \arrow[u, "B^{-1}"'] \end{tikzcd}$$

i.e. 1^{st} column of B^{-1}.

$$\begin{tikzcd} \varphi^* M \arrow[r, "F_0"] \arrow[d, "\varphi^* B"'] & M \arrow[d, "B"] \\ \varphi^* D(G^{univ}) \arrow[r, "F"'] & D(G^{univ}) \end{tikzcd}$$

$$B F_0 = F B^{\varphi} \qquad B = B(u_1)$$

$$\cancel{B = \Rightarrow} \quad B(u_1) = F\, B^{\varphi}(u, p)\, F_0^{-1}$$

$$B(0) = I$$

$$B = F^{a_n} F^{a_{n-1}} \cdots F\, B^{a_{n+1}}(u, p^{n+1})\, F_0^{-(n+1)}$$

$$B = \lim_{n\to\infty} F^{a_n} F^{a_{n-1}} \cdots F\, F_0^{-(n+1)}$$

This is backward

B^{-1} = the matrix we started with

$\Rightarrow$ 1st column of $B^{-1} = \begin{vmatrix} w \\ w w_1 \end{vmatrix}$

Period mappin

$$LT \quad \text{over } \mathbb{W}\langle\langle u_1 \ldots u_{n-1}\rangle\rangle$$

$$\downarrow$$

$$P(M) = \mathbb{P}^{n-1}$$

$B(u_1)$ converges $\nu(u_1) > \frac{1}{p} = r$

$B^{\varphi}(u^p)$ — $\nu(u_1) > \frac{1}{p} r$

$B(u_1)$ converges $\nu(u_1) > 0$

$$LT \qquad\qquad \begin{array}{c} LT_{an} \\ \downarrow \\ \mathbb{P}^{n-1} \end{array}$$

$$H^*(Aut; E_*) \longmapsto ?$$

$$= W[[u_1 \ldots u_{n-1}]][u^{\pm 1}]$$

$$H^*(Aut\,\Gamma; W)$$
$$\downarrow \cong ?$$
$$H^*(Aut\,\Gamma; E_0)$$

(yes $n=2$)

$$\mathrm{Pic}_{Aut\,\Gamma}(LT) = ? \longleftrightarrow \mathrm{Pic}_{Aut\,\Gamma}(LT_{an})$$

$$\begin{tikzcd}
 & \mathrm{GL}_n(\mathbb{Z}_p) \ \mathrm{LT}^{\infty} \arrow[dl] \arrow[dd, dash] \\
\mathrm{LT}_{an} \arrow[dr] & \\
 & \mathbb{P}
\end{tikzcd}$$

$$\mathrm{GL}_n^+(\mathbb{Q}_p)$$

$$\|$$

$$\cancel{M_n(\mathcal{O}_p) \cap \mathrm{GL}.}$$

$$M_n(\mathbb{Z}_p) \cap \mathrm{GL}_n(\mathbb{Q}_p)$$