Last time:

$$\ell(x) = \sum \frac{x^{p^{n}}}{p^n}$$

is the log of a formal gp over $\mathbb{W}$

$$\ell^{-1}(\ell(x)+\ell(y)) \in \mathbb{W}[[x,y]]$$

$$f(x) = \ell(x) + \frac{w_1}{p}\ell(x^p)$$

not the log of a formal gp

$\mathbb{W}[[u_1 \cdots u_{n-1}]] \circlearrowleft \varphi$

$\varphi(u_i) = u_i^p$

$\varphi =$ Frob on $\mathbb{W}$

$$f(x) = x + \frac{u_1}{p} f^{\varphi}(x^p) + \cdots + \frac{u_{n-1}}{p} f^{\varphi^{n-1}}(x^{p^{n-1}}) + \frac{1}{p} f^{\varphi^n}(x^{p^n})$$

$\underline{\underline{n=2}}$

$$f(x) = \sum \underline{m_k}\, x^{p^k}$$

$m_0 = 1$

$m_1 = \frac{u_1}{p}$

$m_2 = \frac{1}{p} + \frac{u_1^{1+p}}{p^2}$

$p \in I \triangleleft A \subset L$

$$\varphi : L \longrightarrow L$$

$$\varphi : A \longrightarrow A$$

$$\varphi(x) \equiv x^p \mod I$$

$$s_1, s_2, \ldots \in L$$

$$\forall_{i,j} \; \varphi^i(s_j) \cdot I \subset A$$

<u>Hazewinkel:</u>

$$f(x) = x + s_1 f^{\varphi}(x^p) + \cdots + s_n f^{\varphi^n}(x^{p^n}) + \cdots$$

Then $f^{-1}(f(x) + f(y)) \in A[[x, y]]$

$$\ell(x) = \sum \frac{x^{p^n}}{p^n}$$

$$\ell(x) = x + \frac{1}{p}\,\ell(x^{p^2})$$

$$f(x) = \ell(x) + \frac{w_1}{p}\,\ell(x^p)$$

If we took $\varphi(w_1) = 0$

$$f(x) = x + \frac{w_1}{p} f^{\varphi}(x^p) + \frac{1}{p} f^{\varphi^2}(x^{p^2})$$

$I = (p)$

$\varphi(w_1) = 0$ need $w_1^p \equiv 0 \ (p)$

$= \frac{w_1^p}{p} = w^{(1)}$

$\varphi(w^{(1)}) = 0$

$\Rightarrow \frac{w^{(1)\,p}}{p}$

$\Longleftrightarrow \forall n \quad \frac{w_1^n}{n!}$

<u>Hazewinkel</u>: $\ell(x) + \frac{w_1}{p}\ell(x^p)$

is the log of a formal gl

over $\mathbb{W}\langle\langle w_1\rangle\rangle$ divided power ring

$\mathbb{W}[[u_1]] \longrightarrow \mathbb{W}\langle\langle w_1\rangle\rangle$

$u_1 \longrightarrow w_1$ mod $p, \ldots$

extends to an iso

$\mathbb{W}\langle\langle u_1\rangle\rangle \longrightarrow \mathbb{W}\langle\langle w_1\rangle\rangle$

Summary:

$E_* = \mathbb{W}[[u_1 \ldots u_{n-1}]][u, u^{-1}]$

$|u| = -2$

Claim over $\mathbb{W}\langle\langle u_1 - u_{n-1}\rangle\rangle[u^{\pm 1}]$

there $w, w_1 \ldots w_{n-1}$

$w \equiv u + \ldots$

$w_i \equiv u_i + \ldots$

st

$$M \longrightarrow \mathbb{W}\langle\langle u_1, \ldots u_{n-1}\rangle\rangle[u^{\pm 1}]$$

$$\gamma \longrightarrow w$$

$$v^i\gamma \longrightarrow w \cdot w_i$$

is <u>equivariant for Aut Γ!</u>

<u>Q</u> Explicitly

$w = ?$

$w w_i = ?$

$n=2$

$$\ell(x) + \frac{w_1}{p}\ell(x^p)$$

$$x + \frac{w_1}{p} x^p + \frac{x^{p^2}}{p^2} + \frac{w_1}{p^2}\frac{x^{p^3}}{p^3} \cdots$$

Lubin-Tate log

$$x + m_1 x^p + m_2 x^{p^2} \cdots$$

$$\Longrightarrow \quad w = \lim_{n \to \infty} p^n m_{2n}$$

$$w w_1 = \lim_{n \to \infty} p^n m_{2n-1}$$

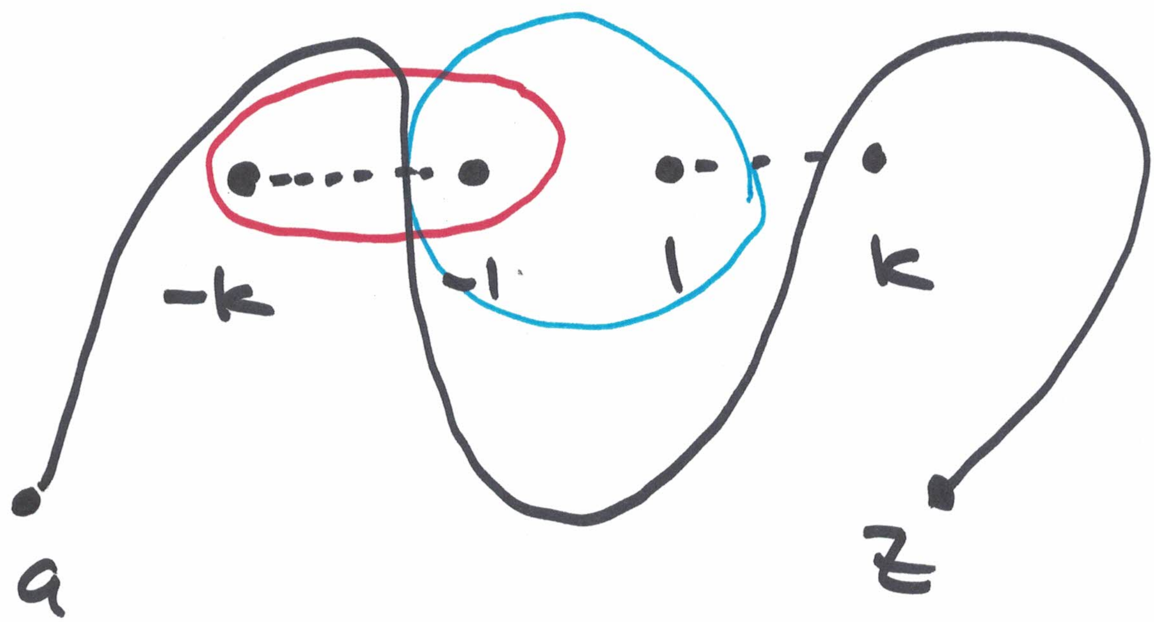
$$A = A[u_1] = \begin{pmatrix} \frac{u_1}{p} & \frac{1}{p} \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega & \frac{1}{p}(\omega w_1)^{\varphi} \\ \omega w_1 & \frac{1}{p}\omega^{\varphi} \end{pmatrix}$$

$$= \lim_{n\to\infty} A^{\varphi^n} \cdots A \cdot A(0)^{-(n+1)}$$

Periods (Classical)

$$\int_a^z \frac{dx}{\sqrt{(x^2-1)(x^2-k^2)}}$$



$$\eta_1 = \int_{red} \qquad \eta_2 = \int_{blue}$$

$\int_a^z \frac{dx}{\sqrt{}}$ is depends only on z in $\mathbb{C}/\langle \eta_1, \eta_2 \rangle$

more generally

X genus g

$\omega_1 \dots \omega_g$ basis of holo. 1-forms

$\gamma_1 \dots \gamma_{2g}$ basis for $H_1(X;\mathbb{Z})$

$$\int_{\gamma_i} \omega_j : H_1(X;\mathbb{Z}) = \Lambda$$
$$\downarrow$$
$$\mathbb{C}^g$$

$$\int_a^z \omega_i : X \hookrightarrow \mathbb{C}^g/\Lambda$$

This can be made to work for formal gps!

$$\begin{tikzcd} H^0(X; \Omega^1) \arrow[r] & H^1_{DR}(X:\mathbb{C}) \arrow[d, leftrightarrow, "\text{rigid}"] \\ & V \end{tikzcd}$$

$\left(\omega_1 - \omega_g \right.$

moving.

Moduli space of X $\longrightarrow$ $Gr_g(V)$

Grassmannian

$X_{\cancel{k}}$ $\qquad k \in K$

$X_k \qquad X \qquad dx, dy, dk$

$$\begin{tikzcd} X \arrow[d] \\ K \end{tikzcd}$$

$k \in K$

$$y^2 = (x^2-1)(x^2-k^2)$$

$\Rightarrow$ connection on $H^{*}_{DR}(X/K)$

Gauss-Manin