CLASSICAL ALGEBRAIC IWASAWA THEORY.

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If one wants to learn Iwasawa theory, the starting point has to be the basic material covered in §1 - §8 of Iwasawa's paper [1]. The aim of these lectures will be to give a concise account of this work, concentrating on proving one of the main results of the paper, which is the so called weak Leopoldt conjecture for the cyclotomic $\mathbb{Z}_p$-extension of any number field, i.e. that the p-adic defect of Leopoldt is always bounded as one goes up the cyclotomic $\mathbb{Z}_p$-extension. The course will also include a brief introduction to the notion of the Iwasawa algebra of $\mathbb{Z}_p$, and the structure theory of modules for this Iwasawa algebra. The background required for the lectures will be basic algebraic number theory, including a knowledge of the main facts of abelian global class field theory.

1. Possible Project

One possible project will be to consider some analogues of the weak Leopoldt conjecture, which remain unproven. For example, if K is any imaginary quadratic field and p is a prime which splits in K into two distinct primes w and $w*$, there is a unique $\mathbb{Z}_p$-extension K_∞/K which is unramified outside the prime w. If F is any finite extension of K, one can then define the compositum of F and K_∞ to obtain a $\mathbb{Z}_p$-extension F_∞/F. There is now an obvious analogue of the Leopoldt conjecture and the Leopoldt defect if one now considers the p-adic closure of the image of the global units in the product of the local units at the primes above w, for any finite layer of F_∞/F. In particular, the analogue of the weak Leopoldt conjecture for F_∞/F should be that this defect of Leopoldt is bounded as one mounts the tower. This question is important in the study of the Iwasawa theory of elliptic curves with complex multiplication [2]. Unfortunately, Iwasawa's proof of weak Leopoldt for the cyclotomic $\mathbb{Z}_p$-extension does not seem to extend to this situation. When F is abelian over K, the p-adic analogue of Baker's theorem proves the w-adic Leopoldt defect is zero for every finite layer of F_∞/F, but only a few examples are known where even the weak w-adic Leopoldt conjecture has been proven once F is not abelian over K. One possible project would be to discuss this material, and extend some of the known non-abelian examples.

References

[1] K. Iwasawa On $\mathbb{Z}_l$-extensions of algebraic number fields, Ann. of Math. 98 (1973), 246-326.

[2] J. Coates *Infinite descent on elliptic curves with complex multiplication*, in Arithmetic and Geometry, Progress in Mathematics 35 (1983), Birkhauser, 107-137.

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