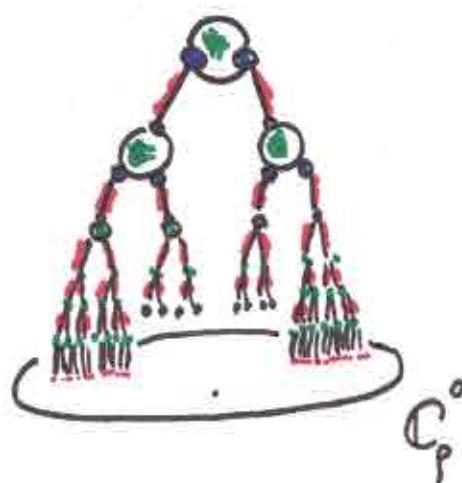


①

# The adic unit disc

$\text{Spa}(\mathbb{Q}_p\langle T \rangle, \mathbb{C}_p^\circ\langle T \rangle)$  - a connected spectral space containing  $\mathbb{C}_p^\circ$

Classification of points:



- (1) Classical ( $\mathbb{C}_p$ -points)
  - (2) Gauß points (rat'l radius)
  - (3) Gauß points (irrat'l radius)
  - (4) Dead ends ( $\mathbb{C}_p$  not sph. comp.)
  - (5) Rank 2 points.
- $\overline{(2)} \cong (5)$ .

## Variations

$$\mathbb{A}_{\mathbb{C}_p}^{1, \text{ad}} = \lim_{\rightarrow} D$$

$$D \subseteq \mathbb{A}_{\mathbb{C}_p}^{1, \text{ad}} \quad \text{open}$$

$$\bar{D} = D \cup \text{pt} = \text{Spa}(\mathbb{Q} A, A^{++})$$

$$\mathbb{P}_{\mathbb{C}_p}^{1, \text{ad}} = \mathbb{A}_{\mathbb{C}_p}^{1, \text{ad}} \cup \{\infty\}$$

② The formal open disc

$$X = \text{Spa}(\mathbb{Z}_p[[T]], \mathbb{Z}_p[[T]]) \xrightarrow{\text{(p,T)-adic}} \text{Spa}(\mathbb{Z}_p, \mathbb{Z}_p) \\ = \{s, \eta\}$$

$$X_\eta = X_\eta = \{x \in X \mid |p(x)| \neq 0\} \rightarrow \text{Spa}(\mathbb{Q}_p, \mathbb{Z}_p) = \{\eta\}$$

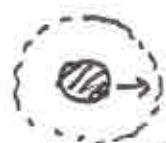
$$\text{If } x \in X_\eta \exists n \gg 0, |\tau(x)^n| \leq |p|$$

$$x \in \{x \in X \mid |\tau(x)|^n \leq |p| \neq 0\} = \bigcup \left( \frac{T^n}{p} \right)$$

$$= \text{Spa} \underbrace{(\mathbb{Z}_p[[T]] [\frac{T^n}{p}]^\wedge [\frac{1}{p}])}_{\left\{ \sum_{n=0}^{\infty} a_n T^n \mid a_n \in \mathbb{Q}_p \text{ conu. on } |T| \leq |p|^{1/n} \right\}}, \quad \text{---}$$

$$\left\{ \sum_{n=0}^{\infty} a_n T^n \mid a_n \in \mathbb{Q}_p \text{ conu. on } |T| \leq |p|^{1/n} \right\}$$

$$X_\eta = \bigcup_{n=1}^{\infty} \bigcup \left( \frac{T^n}{p} \right)$$



not quasi-compact.

$$H^0(X_\eta, \mathcal{O}_{X_\eta}) = \left\{ \sum_{n=0}^{\infty} a_n T^n \mid a_n \in \mathbb{Q}_p \text{ conu. on } |T| < 1 \right\}$$

not Huber ring

It's a Fréchet algebra.

③ From last time:

$$H = \text{formal } G_m$$

$$\begin{aligned} H: \text{complete Huber pairs} &\longrightarrow \mathbb{Z}_p\text{-modules} \\ (R, R^+) &\longmapsto 1 + R^{\circ\circ} \text{ under mult.} \\ /(\mathbb{Z}_p, \mathbb{Z}_p) &\qquad\qquad\qquad \uparrow \text{top nilpotent elts} \end{aligned}$$

$$H = \text{Spa}(\mathbb{Z}_p \parallel T \parallel, \mathbb{Z}_p \parallel T \parallel)$$

$$\begin{aligned} H(R, R^+) &= \text{Hom}(\mathbb{Z}_p \parallel T \parallel, R^+) = R^+ \cap R^{\circ\circ} \\ &= R^{\circ\circ} \end{aligned}$$

$H_{\mathbb{Q}_p}$  is  $\mathbb{Z}_p$ -mod. object in category of adic spaces  $/\mathbb{Q}_p$ .

$$\text{Let } \tilde{H} = \varprojlim_p H \quad \begin{matrix} \text{"universal cover"} \\ \nearrow \mathbb{Q}_p\text{-vector space} \end{matrix}$$

$$\tilde{H}(R, R^+) = \varprojlim_{x \mapsto x^p} 1 + R^{\circ\circ}$$

$$\xrightarrow{\sim} \varprojlim_{x \mapsto x^p} 1 + R^{\circ\circ}/p \simeq \varprojlim_{x \mapsto x^p} R^{\circ\circ}/p \simeq \varprojlim_{x \mapsto x^p} R^{\circ\circ}$$

$(p, T)$ -adically complete

$$\tilde{H} = \text{Spa}(\mathbb{Z}_p \llcorner T^{1/p^\infty}], \mathbb{Z}_p \llcorner T^{1/p^\infty}])$$

For a perfectoid field  $K$ ,  $\tilde{H}_K$  is a perfectoid space

(4)

Last time:  $\tilde{H}(C^\circ) = H(C^\circ) = 1 + m_C$

$C/\mathbb{F}_p$  alg.  
closed perf.  
field.

 $\tilde{H}(C^\circ)/\mathbb{Z}_p^* \cong \{\text{units of } C\}$

If  $C^\#$  is an unit:  $\tilde{H}(C^{\#0}) \cong \tilde{H}(C^\circ)$

$$0 \rightarrow \mu_{p^\infty}(C^\#) \rightarrow \tilde{H}(C^{\#0}) \xrightarrow{\log} C^\# \rightarrow 0$$

apply  $\varprojlim_p$

exact  
seq of  
 $\mathbb{Z}_p$ -modules:

$$0 \rightarrow \mathcal{Q}_p(1) \rightarrow \tilde{H}(C^{\#0}) \rightarrow C^\# \rightarrow 0$$

$\tilde{H}(C^\circ)$

" of  
 $\mathbb{Q}_p$ -vector  
spaces

Colmez: Banach Space of dimension  
(1, 1)

Fargues-Fontaine curve.

(5)

$C/\mathbb{F}_p$  alg. closed perfectoid field  $C \ni \omega$

units of  $C \longleftrightarrow$  ~~etale~~ ideals  $(\xi) \subseteq W(C^\circ)$   
of char 0  $\xi$  is primitive deg 1.

For  $C^\#$  unit of  $C$   
~~etc~~

$$\theta_{C^\#}: W(C^\circ)[\frac{1}{p[\omega]}] \rightarrow C^\#$$

$$[x] \mapsto x^\#$$

Def (adic FF curve)

$$Y_C = \text{Spa}(W(C^\circ), W(C^\circ)) \setminus \{x \mid \wp(p[\omega])(x) \in \{p[\omega] = 0\}\}$$

{units of}  $\rightarrow Y_C$ .

$C^\#$   
to char 0

Thm (kedlaya)  $Y_C$  is an adic space

$$B_C := H^0(Y_C, \mathcal{O}_{Y_C}) \quad \text{Fréchet algebra.}$$

(6)

Given

$$\dot{\varepsilon} \in \tilde{H}(C^\circ) \simeq 1 + m_C, \text{ get}$$

$$[\varepsilon] \in H(W(C^\circ))$$

$$[\varepsilon]^\phi = [\varepsilon^p] = [\varepsilon]^p$$

$$t = \log [\varepsilon] = \sum (-D^{n-1}) \frac{([\varepsilon] - 1)^n}{n} \in B_C$$

(≈ \xi)

$$t^\phi = \log [\varepsilon]^\phi = \log [\varepsilon]^p$$

$$= p t.$$

$$t \in B_C^{\phi=p} \quad x \in C^\circ$$

\Downarrow

$$\dots + p^2[x^{1/p^2}] + p[x^{1/p}] + (x) + \frac{[x^p]}{p} + \frac{[x^{p^2}]}{p^2} + \dots$$

Thm. (Fargues-Fontaine)

$$\tilde{H}(C^\circ) \xrightarrow{\sim} B_C^{\phi=p}$$

If  $C^\#$  is an untilt to char 0,

$\theta_{C^\#}: W(C^\circ) \rightarrow C^\#$  extends to  $B_C$

$$0 \rightarrow \mathbb{Z}_p(G) \rightarrow \tilde{H}(C^\circ) \xrightarrow{\log} C^\# \rightarrow 0$$

$\cong \downarrow \quad |? \quad ||$

$$0 \rightarrow \mathbb{Z}_p t \rightarrow B_C^{\phi=p} \xrightarrow{\theta_{C^\#}} C^\# \rightarrow 0$$

Variation  $d, h > 1$  rel prime  $d \leq h$  (7)

$H_{d/h}$  formal group  $b + h$  dim  $d/\mathbb{Z}_p$   
eg  $E$  ss.  $\sim \hat{E}_0 = H_{1/2}$ .

$H_{d/h} \simeq \text{Spf } \mathbb{Z}_p[[T_1, \dots, T_d]]$

$\tilde{H}_{d/h} \simeq \text{Spf } \mathbb{Z}_p[[T_1^{1/p^\infty}, \dots, T_d^{1/p^\infty}]]$

$\tilde{H}_{d/h, K}$  is a  $\mathbb{Q}_p$ -univ. object in  
cat. of perf spaces

$0 \rightarrow \mathbb{Q}_p^h \rightarrow \tilde{H}_{d/h}(C^\circ) \rightarrow (C^\#)^d \rightarrow 0$

$$B_C^{\phi^h = p^d} \rightsquigarrow$$

~~0~~