R = Grothendieck ring of varieties$/k$

(additively generated by $[X]$ ← variety,

additive relations gen. by $[X] = [U] + [Z]$,

multiplication gen. by $[X] \times [Y] = [X \times Y]$.

Is $\mathbb{L}$ 0-divisor? $R \hookrightarrow R_{\mathbb{L}}$

Is R domain? Not!

Kollár Poonen

$/\mathbb{C}$

nonisomorphic abelian varieties

$$A \times A = B \times B$$

$$\underbrace{([A]+[B])}_{\neq 0 = [\phi]} \underbrace{([A]-[B])}_{I} = 0$$

$$X \xrightarrow{\text{Alb}} Y \text{ ab. varieties.}$$

Alb(X).

Depends only stable birationality class.

$$R \underset{\checkmark}{\longrightarrow} \mathbb{Z}[SB] \underset{\checkmark}{\longrightarrow} \mathbb{Z}[AV]$$

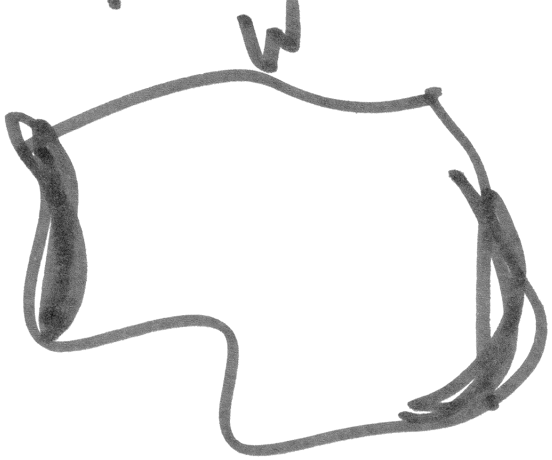
X, Y varieties

~~$[X, Y]$~~

$[X] = [Y]$ $\Leftarrow$ X, Y "piecewise iso" ?!

$[Bl_p \mathbb{P}^2] = [\mathbb{P}^1 \times \mathbb{P}^1]$

Piecewise Isomorphism



Thm (Q. Liu, Sebag). X, Y varieties $K = \bar{\mathbb{Q}}$ $\mathbb{C}$. Then if $[X] = [Y]$ and one holds: (i) $\dim X = 1$

(ii) smooth proper surface

(iii) X contains finite # rational curves.

then X can be cut-and-pasted to Y!

<u>Recall:</u> $\hat{R}_{\mathbb{L}} = \lim R[1/\mathbb{L}]$

$$\hat{R} = \varprojlim R/\mathbb{L}^n$$

$$\uparrow$$

R (Is $(\mathbb{L}^n) = (0)$)

<u>Properties</u>

- pt-counting works.
- (L) kernel of $R \to \mathbb{Z}[SB]$

$$\hat{R} \longrightarrow \mathbb{Z}[SB]$$

• $(\hat{R})_{\mathbb{L}} \longrightarrow \widehat{(R_{\mathbb{L}})}$ surjection

$\mathbb{L}$ not 0-divisor: is iso.

<u>Thm</u> (D. Litt). Suppose X smooth irred. proj surface with $h^0(X, \omega_X) \neq 0$. Then for any $m > 0$, $\mathrm{Sym}^m X$ <u>not</u> stably birat'l to $\mathrm{Sym}^n X$ for $n \gg 0$.

Cor For such X

$\varprojlim \mathrm{Sym}^n X$ does not exist in $\hat{R}$.

Cor $\mathbb{L}$ not 0-div. $(\hat{R}_{\mathbb{L}} \supseteq \hat{R}_{\mathbb{L}})$

MSSP false.

Motivic Stab. of Sym. Pow

$$\boxed{\frac{\mathrm{Sym}^n X}{\mathbb{L}^{n \dim X}}} \quad \hat{R}_{\mathbb{L}} \quad R[[\tfrac{1}{\mathbb{L}}]]$$

Cor Piecewise conj. contradicts MSSP

$$\frac{[X]}{\mathbb{L}^{\cdot}} = [Y]/\mathbb{L}^{\cdot}$$

Lev Borisov arXiv: 1412.6194.

Thm $\mathbb{L}$ is zero-divisor

Piecewise Isomorphism. Conj wrong.

$$([X]-[Y])(\mathbb{L}^2-1)(\mathbb{L}-1)\mathbb{L}^7=0$$

CY

Pfaffian - Grassmannian double mirror correspondence.

Stabilization in Gr. Ring

points on a line. $\mathbb{A}^1$.

Space of unordered pts on $\mathbb{A}^1$

$$\mathbb{C}\{x^n + a_{n-1}x^{n-1} + \cdots + a_0\} = \mathbb{A}^n$$

$$\mathrm{Sym}^n \mathbb{A}^1 = \mathbb{A}^1$$

$$\Delta_\nu \qquad \nu \mapsto n$$

$$\overline{\Delta_{m\,1^{n-m}}}$$

Jumping-off point: prob. of not having m-fold root? (or worse)

<u>arithmetic</u> $k = \mathbb{F}_q$. $(n \geq m)$

Ans: $1 - \dfrac{\# \bar{\Delta}_{1^{n-m}m}}{q^n} = 1 - \dfrac{1}{q^{m-1}}$

TOP 1969.

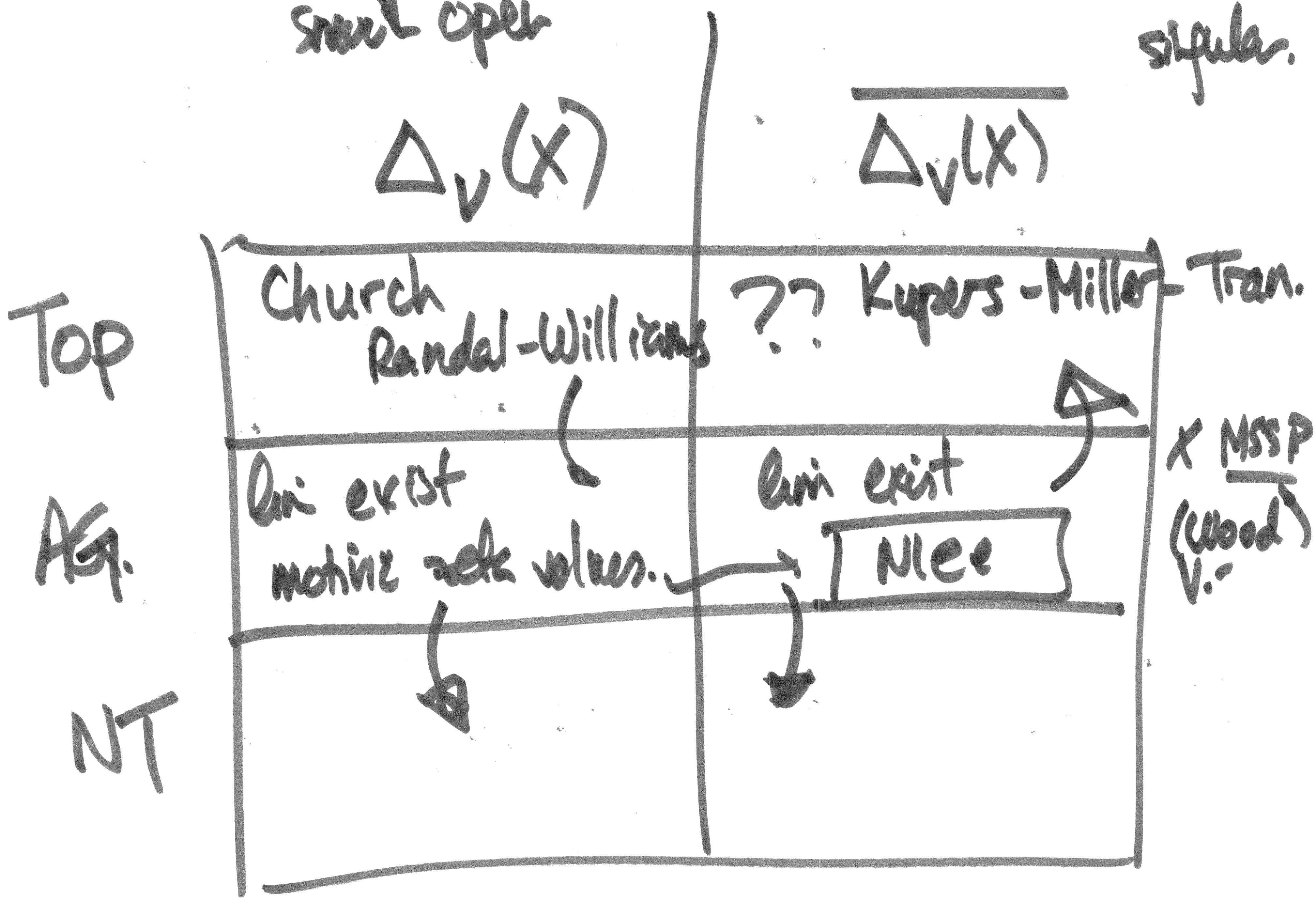
$k = \mathbb{C}$

Thm (Arnol'd+) $h^i(\mathbb{A}^n \setminus \Delta_m) =$

$$\begin{cases} 1 & \text{if } i = 0,\ i = 2m-1 \\ 0 & \text{otherwise.} \end{cases}$$

AG. cut-and-paste.

$$[\Delta_{m\,1^{n-m}}] = \mathbb{A}^{n-(m-1)}$$



smooth open
singular.
$\Delta_V(X)$
$\overline{\Delta_V(X)}$
Top
AG.
NT
Church
Randal-Williams
??
Kupers-Miller
Tran.
lim exist
motivic zeta values.
lim exist
Nlee
X MSSP
(Wood V.)

$\overline{\Delta}_{ab} \, 1^{n-a-b}$ NT $\frac{1}{q^{a+b-2}}$ Fq.

AG. Δ

TOP.

$\overline{\Delta}_{223} \, 1^{n-7} \mapsto \frac{1}{q} + \cdots$

n points on X. What are $(n \to \infty)$
odds that there are exactly
m multiple points → n split into exactly m parts.

$$\lim = \frac{\cdots + \mathrm{Sym}\left(\frac{1}{|\mathbb{I}|^m}\right)^n + \cdots}{Z_X(1/\mathbb{I}^{2d}) = 1 + \cdots + \mathrm{Sym}^n X\left(\frac{1}{|\mathbb{I}|^n}\right)^n + \cdots}$$

$m=0$ $\quad \dfrac{1}{Z(\;)}$ $\quad m=1$