# Group actions on curves and the lifting problem
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## 1 Outline

**1.1 The lifting problem** The problem we are concerned with in our lectures and which we shall refer to as the *lifting problem* was originally formulated by Frans Oort in [17]. To state it, we fix an algebraically closed field $\kappa$ of positive characteristic $p$. Let $W(\kappa)$ be the ring of Witt vectors over $\kappa$. Throughout our notes, $\mathfrak{o}$ will denote a finite local ring extension of $W(\kappa)$ and $k = \mathrm{Frac}(\mathfrak{o})$ the fraction field of $\mathfrak{o}$. Note that $\mathfrak{o}$ is a complete discrete valuation ring of characteristic zero with residue field $\kappa$.

**Definition 1.1** Let $C$ be a smooth proper curve over $\kappa$. Let $G \subset \mathrm{Aut}_\kappa(C)$ be a finite group of automorphisms of $C$. We say that the pair $(C, G)$ *lifts to characteristic zero* if there exists a finite local extension $\mathfrak{o}/W(\kappa)$, a smooth projective $\mathfrak{o}$-curve $\mathcal{C}$ and an $\mathfrak{o}$-linear action of $G$ on $\mathcal{C}$ such that

(a) $\mathcal{C}$ is a lift of $C$, i.e. there exists an isomorphism $\lambda : \mathcal{C} \otimes_\mathfrak{o} \kappa \cong C$, and

(b) the $G$-action on $\mathcal{C}$ restricts, via the isomorphism $\lambda$, to the given $G$-action on $C$.

**Problem 1.2 (The lifting problem)** Which pairs $(C, G)$ as in Definition 1.1 can be lifted to characteristic zero?

**Remark 1.3** It is easy to find pairs $(C, G)$ which cannot be lifted to characteristic zero. To see this, assume that $(\mathcal{C}, G)$ is an equivariant lift of $(C, G)$. Then both the special fiber $C$ and the generic fiber $\mathcal{C}_k := \mathcal{C} \otimes_\mathfrak{o} k$ of $\mathcal{C}$ are smooth projective curves of the same genus $g$. Since the characteristic of $k$ is zero, the classical *Hurwitz bound* applies and shows that for $g \geq 2$ we have

$$|G| \leq 84(g-1). \tag{1}$$

See e.g. [6], Exercise IV.2.5. However, in characteristic $p$ there exist pairs $(C, G)$ violating this bound (see Exercise 1.11 for an example). It follows that such pairs $(C, G)$ cannot be lifted.

Another way to produce counterexamples is to take $C := \mathbb{P}^1_\kappa$ and use that $\mathrm{Aut}_L(\mathbb{P}^1_L) = \mathrm{PGL}_2(L)$ for any field $L$, see Exercise 1.10

**1.2 The local lifting problem and the local-global principle** At first sight, Remark 1.3 seems to suggest that the liftability of a pair $(C, G)$ is a global issue, as the Hurwitz bound depends on the genus. However, Theorem 1.5 below states that, on the contrary, liftability depends only on the (finite) set of closed points $y \in C$ with nontrivial stabilizers $G_y \subset G$ and the action of $G_x$ on the formal neighborhood of $y$. The lifting problem is thus reduced to the following *local lifting problem*.

**Definition 1.4** A *local action* is a pair $(\bar{A}, G)$, where $\bar{A} = \kappa[[z]]$ is a ring of formal power series in $\kappa$ and $G \subset \mathrm{Aut}_\kappa(\bar{A})$ is a finite group of automorphisms of $\bar{A}$. We say that the pair $(\bar{A}, G)$ *lifts to characteristic zero* if there exists a finite extension $\mathfrak{o}/W(k)$ and an action of $G$ on $A := \mathfrak{o}[[z]]$ lifting the given $G$-action on $\bar{A}$.

The *local lifting problem* is the question 'which local actions $(\bar{A}, G)$ lift to characteristic zero?'.

**Theorem 1.5 (The local-global principle)** *Let $(C, G)$ be as in Definition 1.1. Then $(C, G)$ lifts if and only if for all closed points $y \in C$ the induced local action $(\hat{\mathcal{O}}_{C,y}, G_y)$ lifts. (Note that $\hat{\mathcal{O}}_{C,y}$ is a ring of formal power series since $C$ is smooth over $\kappa$.)*

**Proof:** One direction is more or less obvious: if $(\mathcal{C}, G)$ is a lift of $(C, G)$, then smoothness of $\mathcal{C}$ shows that $\hat{\mathcal{O}}_{\mathcal{C},y}$ is a ring of formal power series over $\mathfrak{o}$. Therefore, $(\hat{\mathcal{O}}_{\mathcal{C},y}, G_y)$ is a lift of $(\hat{\mathcal{O}}_{C,y}, G_y)$, for all $y \in C$.

For the proof of the converse, see e.g. [3], [2], or [1]. □

**Corollary 1.6** *Suppose that for all $y \in C$ the order of the stabilizer $G_y$ is prime to $p$. Then $(C, G)$ lifts.*

**Proof:** Assume that $p \nmid |G_y|$. Then Exercise 1.12 (i) shows that $G_y$ is cyclic. Moreover, one can choose $z \in \hat{\mathcal{O}}_{C,y}$ such that $\hat{\mathcal{O}}_{C,y} = \kappa[[z]]$ and $\sigma(z) = \bar{\zeta} \cdot z$ (here $\sigma$ is a generator of $G$ and $\zeta \in \kappa$ a primitive $n$th root of unity). Since $(p, n) = 1$, Hensel's Lemma shows that $\bar{\zeta}$ lifts uniquely to an $n$th root of unity $\zeta \in \mathfrak{o}$. So the rule $\sigma(z) := \zeta \cdot z$ defines a lift of the natural $G_y$-action on $\hat{\mathcal{O}}_{C,y} = \kappa[[z]]$ to the ring $\mathfrak{o}[[z]]$. Now apply Theorem 1.5. □

**Remark 1.7** Corollary 1.6 corresponds to a well known fact from Grothendieck's theory of the *tame fundamental group*, see [5]. Let $D := C/G$ denote the quotient curve and $x_1, \ldots, x_r \in D$ the images of the points on $C$ with nontrivial stabilizers. Then the quotient map $f : C \to D$ is a $G$-Galois cover, which is tamely ramified in $x_1, \ldots, x_r$ under the hypothesis of Corollary 1.6. Let $\mathcal{D}$ be a lift of $C$ to a smooth proper $\mathfrak{o}$-curve (which exists by [5], Chapter 3). Choose sections $x_{\mathfrak{o},i} : \mathrm{Spec}\, \mathfrak{o} \to \mathcal{D}$ lifting the points $x_i$. Now Grothendieck's theory shows that there exists a *unique* lift of the cover $\pi$ to a $G$-Galois $f_{\mathfrak{o}} : \mathcal{C} \to \mathcal{D}$ tamely ramified along the sections $x_{\mathfrak{o},i}$. By construction, $(\mathcal{C}, G)$ is a lift of $(C, G)$.

The standard proof of this result (and of Theorem 1.5) uses *formal patching* (see e.g. the lectures by Hartmann and Harbater).

**Remark 1.8** Let $G$ be a finite group of automorphisms either of $\kappa[[z]]$ or of $\mathfrak{o}[[z]]$. Then $G$ is a so-called *cyclic-by-p group*, i.e. $G = P \rtimes C$, where $P$ is the Sylow $p$-subgroup of $G$ and $C$ is a cyclic group of order prime to $p$ (see Exercise 1.12).

This results significantly cuts down the classes of groups we have to consider for the local lifting problem. But it does not give any obstruction against liftability, because it applies to both rings $\kappa[[z]]$ and $\mathfrak{o}[[z]]$.

**1.3 Group actions versus Galois covers** It turns out to be extremely difficult to approach the lifting problem by working directly with automorphisms of $\kappa[[z]]$ and $\mathfrak{o}[[z]]$ in terms of explicit power series (see Exercise 1.13). To really get our hands on the problem we need a shift of perspective.

Let $C$ be a smooth projective curve over $\kappa$ and $G \subset \mathrm{Aut}_\kappa(C)$ a finite group of automorphisms. We have already remarked that the quotient map $f : C \to D := C/G$ is a finite Galois cover. Knowing the pair $(C, G)$ is equivalent to knowing the cover $f : C \to D$, and in principal we can replace *group actions* by *Galois covers* everywhere.

An advantage of this point of view is that we have more tools to construct these objects. For instance, to construct a Galois cover $f : C \to D$ of a given curve $D$ it suffices to define a Galois extension $L/K$ of the function field $K := \kappa(D)$; the corresponding curve $C$ is then simply the normalization of $C$ in $L$. The same approach works in the local setting, see Exercise 1.14.

Lifting group actions is also equivalent to lifting Galois covers. However, the two points of view may lead to very different techniques for solving instances of the lifting problem. For instance, if one proves the Local-Global-Principle (Theorem 1.5) with formal patching (as e.g. in [2]) one uses the perspective of covers. In contrast to this, the proof given in [1] works directly with a pair $(C, G)$.

**1.4 Stable models of Galois covers and Hurwitz trees** We try to describe, as briefly as possible, our approach to the lifting problem, which is based on the study of semistable reduction and group actions on semistable curves. The origin of this method is the work of Raynaud, Green, Matignon and Henrio ([4], [7]).

We start with a Galois cover $f : Y \to X$ of smooth projective curves over the local field $k$. (Recall that $k$ is a complete discrete valuation field of characteristic zero, whose residue field has characteristic $p > 0$.) Let $G$ denote the Galois group of $f$. Let $y_1, \ldots, y_r$ be the ramifications points of $f$ (we assume that all of them are $k$-rational). Assuming that $2g(Y) - 2 + r > 0$ and that $k$ is sufficiently large, there exists a canonically defined $\mathfrak{o}$-model $\mathcal{Y}$ of $Y$ called the *stably marked model*. It is the minimal $\mathfrak{o}$-model of $Y$ which is semistable and such that the points $y_i$ specialize to pairwise distinct smooth points on the special fiber $\bar{Y} := \mathcal{Y} \otimes_{\mathfrak{o}} \kappa$. The action of $G$ on $Y$ extends to $\mathcal{Y}$. The quotient scheme $\mathcal{X} := \mathcal{Y}/G$ is a semistable model of $X$. We call the quotient map $\mathcal{Y} \to \mathcal{X}$ the *stable model* of the Galois cover $f : Y \to X$ and its restriction to the special fiber $\bar{f} : \bar{Y} \to \bar{X}$ the *stable reduction* of $f$.

We say that the cover $f$ has *tame good reduction* if $\bar{Y}$ and $\bar{X}$ are smooth. If this is the case, then $\bar{X}$ and $\bar{Y}$ are irreducible and $\bar{f}$ is an at most tamely ramified $G$-Galois cover. We can consider $f$ (resp. the pair $(Y, G)$) as a lift of $\bar{f}$ (resp. of the pair $(\bar{Y}, G)$). Grothendieck's theory of the tame fundamental group (compare Corollary 1.6 and Remark 1.7) says that the lift $f$ is uniquely determined by $\bar{f}$, the curve $X$ and the choice of points $x_j \in X$ lifting the branch points of $\bar{f}$.

We are of course more interested in the case that $f$ has bad reduction. Then the map $\bar{f}$ is still a finite $G$-invariant map, but it will typically reveal phenomena of wild ramification. Firstly, if $\bar{Y}_1 \subset \bar{Y}$ is an irreducible component, then the restriction of $\bar{f}$ to $\bar{Y}_1$ is in general inseparable. Even if it is separable, it may be wildly ramified.

**Problem 1.9** Can one characterize the finite $G$-invariant maps $\bar{f} : \bar{Y} \to \bar{X}$ between semistable curves over $\kappa$ which arise as the stable reduction of a $G$-Galois cover $f : Y \to X$?

In some sense, the lifting problem is just a special case of this problem, for the following reason. Suppose $\bar{f}_0 : \bar{Y}_0 \to \bar{X}_0$ is a $G$-Galois cover between smooth curves over $\kappa$. Let $f : Y \to X$ be a lift of $\bar{f}$, defined over the local field $k$. Let $\bar{f} : \bar{Y} \to \bar{X}$ be the stable model of $f$. Then $\bar{f}_0$ can be recovered from $\bar{f}$ by contracting all but one component of $\bar{Y}$ and $\bar{X}$. More precisely, we have a commutative diagram

$$\begin{array}{ccc} \bar{Y} & \longrightarrow & \bar{Y}_0 \\ {\scriptstyle \bar{f}}\big\downarrow & & \big\downarrow{\scriptstyle \bar{f}_0} \\ \bar{X} & \longrightarrow & \bar{X}_0, \end{array}$$

where the horizontal arrows are contraction maps. In this situation we will say that the cover $f$ has *good reduction* (as opposed to tame good reduction). In the light of Problem 1.9 we regard $\bar{f}$ as an 'enhancement' of $\bar{f}_0$ which encodes information on the lift $f$. Thus, in order to show that a lift of $\bar{f}_0$ exists it is natural to first try to enhance $\bar{f}_0$ to a map $\bar{f}$ with certain good properties and then try to lift $\bar{f}$.

In the stated generality, Problem 1.9 is very hard. If $p$ does not divide the order of $G$, a complete answer is known by the theory of *tame admissible covers*. The only other case that is reasonably well understood is when the Sylow $p$-subgroup of $G$ has order $p$, and we will often make this assumption during our lectures. But the general philosophy can be described, without any assumption, as follows. Using *higher ramification theory* we attach to the map $\bar{f}$ certain extra data (called *Swan conductors*, *deformation data* and the like). We then try to find enough rules that these extra data must satisfy (with respect to the map $\bar{f}$, the action of $G$ etc.) Finally, we try to show that any map $\bar{f}$ together with enough extra data satisfying all known rules occur as the stable reduction of a Galois cover in characteristic zero.

**1.5 Content and focus of our lectures** In our lectures we do not try to give a comprehensive survey of known results on the lifting problem. Instead, we focus on a few particular results (both positive and negative) and the methods they rely on. Our choice of results and the way we present them will be very much biased by our own contributions to the topic.

It is sometimes useful to distinguish between negative and positive results. Here we call a result *negative* if it gives some obstruction against liftability which shows that certain pairs $(C, G)$ (resp. $(\bar{A}, G)$) cannot be lifted. A result is called *positive* if it shows that certain pairs can be lifted. In these notes, however, we treat both aspects simultaneously, and stress the principals underlying positive and negative results. We plan to treat the following topics in some detail.

- **Obstructions:** A systematic way to find necessary conditions for liftability of a pair $(\bar{A}, G)$ is to study group actions on semistable curves. Using ramification theory, we can attach certain invariants to such an action which 'live on the special fiber'. Compatibility rules connecting these invariants then lead to necessary conditions for liftability of local actions $(\bar{A}, G)$. These can be roughly classified into three types, which we call *combinatorial*, *metric* and *differential*. We will focus on the general approach and on some specific but enlightening examples.

- **$p$-Sylow of order $p$:** Let $(\bar{A}, G)$ be a local action such that $p$ strictly divides the order of $G$ (then $G \cong \mathbb{Z}/p\mathbb{Z} \rtimes \mathbb{Z}/n\mathbb{Z}$, with $(p, n) = 1$ by Remark 1.8). In this case there is an if-and-only-if condition for liftability which only depends on certain discrete invariants attached to $(\bar{A}, G)$. In other words, $(\bar{A}, G)$ lifts if and only if the *Bertin obstruction* vanishes. This was proved in [9] and [8]. We will explain the main steps of the proof.

- **Cyclic actions and the Oort conjecture:** It is expected that for a cyclic group $G$ all local actions $(\bar{A}, G)$ lift. This expectation is traditionally called the *Oort conjecture*. It has been proved for cyclic groups of order $p^n m$, where $(p, m) = 1$ and $n \leq 3$ (see [18] for $n = 1$, [3] for $n = 2$ and [14] for $n = 3$). We will present some elements of the recent work of Obus and the second author which gives a sufficient condition for liftability of general cyclic actions and in particular settles the case $n \leq 3$ of the Oort conjecture.

**1.6 Prerequisites and reading list** We will assume that students are more or less familiar with the following material.

- Artin–Schreier and Kummer Theory, Witt vectors, Hensel's Lemma. These can be found in most algebra book on the graduate level. A more advanced approach to Artin–Schreier and Kummer Theory can be found in [12].

- Ramification theory of local fields, in particular higher ramification groups and conductors. The standard reference in [15], Chapters 3–5. Section 2 of our notes will contain a review of all the results that we will need. These include the case of a non-perfect residue field (see [19] and the course notes of Saito and Mieda), but we will not assume that this material is already known.
- Blowing-up, arithmetic surfaces, models of curves. The definition of blowing-up can for example be found in [11], Section 8.1. Section 10.1 of [11] contains more material on arithmetic surfaces than we will require. Concrete examples of blowing-ups of arithmetic surfaces can for example be found in [16], Chapter IV.
- The $p$-adic disk (this is the rigid analytic spaces associated to the ring of power series $\mathfrak{o}[[z]]$). We will not use rigid analysis in any deep way, but you should know at least some basic facts about power series over $p$-adic rings, such as the Weierstrass Preparation Theorem and properties of the Newton polygon. See e.g. [10]. Their rigid-geometric interpretation will be explained in our notes.

For a recent overview on the lifting problem we recommend [13].

**1.7 Project description** The goal of our project is to solve the local lifting problem for the group $A_4$ when $p = 2$. This result was announced in [9], but the proof has never been written up. Note that the Sylow 2-subgroup of $A_4$ has order 4, and hence this case is not covered by the results of [9] and [8].

There are several steps involved in this project which may be treated separately. Most of the material explained during the lectures will appear at some stage of the project. (The strange terms occurring in the following description will be explained in detail in our notes.)

(a) Classify all local $A_4$-action $A_4 \subset \mathrm{Aut}_\kappa(\kappa[[z]])$ over an algebraically closed field $\kappa$ of characteristic 2 in terms of filtration of higher ramification groups (or equivalently, in terms of the Artin conductor).

(b) Show that the Bertin Obstruction vanishes for every local $A_4$-action.

(c) Construct Hurwitz trees for every local $A_4$-action.

(d) Show that every Hurwitz tree constructed in (c) can be lifted to an $A_4$-action in characteristic zero.

If the project succeeds, there is the possibility of expanding the result into a publishable paper.

**1.8 Exercises** Here are some warm up exercises which should be helpful to get started.

**Exercise 1.10** Prove the following statements (which give an example of a pair $(X, G)$ that does not lift).

(a) For any field $K$ the group of automorphisms of $\mathbb{P}^1_K$ is $\mathrm{PGL}_2(K)$.

(b) Let $\kappa$ be an infinite field of characteristic $p$. Then there exists a subgroup $G \subset \mathrm{PGL}_2(\kappa)$ isomorphic to $(\mathbb{Z}/p\mathbb{Z})^n$, for all $n \geq 1$.

(c) If $K$ is a field of characteristic zero, and $G \subset \mathrm{PGL}_2(K)$ is isomorphic to $(\mathbb{Z}/p\mathbb{Z})^n$, then $n \leq 1$ or $p^n = 4$.

**Exercise 1.11** Let $X$ be the smooth projective curve over $\kappa := \bar{\mathbb{F}}_p$ with affine plane model

$$y^p - y = x^{p+1}.$$

(a) Compute the genus of $X$ (e.g. by using the Riemann–Hurwitz formula).

(b) Fix a primitive $(p^2-1)$th root of unity $\zeta \in \kappa$ and let $\sigma, \tau \in \mathrm{Aut}_\kappa(X)$ be the automorphisms given by

$$\sigma^*(x) = x, \quad \sigma^*(y) = y + 1$$

and

$$\tau^*(x) = \zeta \cdot x, \quad \tau^*(y) = \zeta^{p+1} \cdot y.$$

Compute the order of the subgroup $G \subset \mathrm{Aut}_\kappa(X)$ generated by $\sigma$ and $\tau$. Show that $G$ violates the Hurwitz bound (1) for $p \gg 0$.

**Exercise 1.12** (a) Let $G \subset \mathrm{Aut}_\kappa(\kappa[[z]])$ be a finite group of automorphisms of $\kappa[[z]]$. Then $G$ is cyclic-by-$p$, i.e. of the form $G = P \rtimes H$, where $P$ is the Sylow $p$-subgroup of $G$ and $C$ is cyclic of order prime to $p$. (This result is well-know and can for example be found in [15].)

Show that if $\sigma \in G$ is of order prime to $p$ there exists a parameter $z' = z + a_2 z^2 + \ldots$ such that $\sigma(z') = \zeta \cdot z'$, where $\zeta \in \kappa$ is a root of unity.

(b) Now let $G \subset \mathrm{Aut}_{\mathfrak{o}}(\mathfrak{o}[[z]])$ be a finite subgroup. Prove the same statement as in (a).

(c) Verify (a) for the nontrivial local actions induced by the examples in Exercise 1.10 and 1.11.

**Exercise 1.13** (a) Show that the automorphism $\sigma : \kappa[[z]] \xrightarrow{\sim} \kappa[[z]]$ given by

$$\sigma(z) := z/(1+z)$$

has order $p$.

(b) Assume $p = 2$ or $p = 3$. Lift the automorphism $\sigma$ in (a) to an automorphism of order $p$ of $\mathfrak{o}[[z]]$, for a suitable ring extension $\mathfrak{o}/W(\kappa)$.

**Exercise 1.14** Fix $h \in \mathbb{N}$, $(h, p) = 1$. Set $A := \kappa[[t]]$ and $K := \mathrm{Frac}(A)$. Let $L := K(y)$ be the Galois extension given by the Artin-Schreier equation

$$y^p - y = t^{-h}.$$

Let $B$ be the integral closure of $A$ in $L$.

(a) Find $z \in B$ such that $B = \kappa[[z]]$.

(b) Let $\sigma \in \mathrm{Gal}(L/K) \cong \mathbb{Z}/p\mathbb{Z}$ be the generator with $\sigma(y) = y + 1$. Determine $z' := \sigma(z) \in B$ as a power series in $z$.

(c) Compare with the automorphism $\sigma$ from Exercise 1.13 (a).

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