1-
17
2- -

$\underline{CH_0(S)}$ Chow gp of 0-cycles

$$= \left(\bigoplus_{s \in S} \mathbb{Z} \cdot [s] \Big/ \underline{\text{rational equivalence}} \right) \Big/ \mathbb{Q}(\zeta_{p^\infty})$$

s : closed pts.

$\cup$

$$\underline{Sw_U(\mathcal{F})} = \sum_{\sigma \in G} \underline{S_G(\sigma)} \cdot \mathrm{Tr}(\sigma : V)$$

V: ℓ-adic rep'n of $\pi_1(U, \bar{x})$ corresp. to $\mathcal{F}$

[Simplifying assump'n

$\pi_1 \to G \curvearrowright \underline{V}$

G: finite gp; V: $\overline{}$ rep'n

$$\begin{tikzcd} V \arrow[r, hook, "\text{cpt'n}"'] \arrow[d, "G"'] & Y \arrow[d] \\ U \arrow[r, hook, "\text{cpt'n}"'] & X \end{tikzcd}$$

< Sch

finite étale
Galois covering

1–18
2–

Simplifying assum'p.

Y smooth dim d

$V = Y \setminus D$

divisor of Y with
simple normal crossing

$D = \bigcup_i D_i$. smooth

$D_{i_1} \cap \cdots \cap D_{i_j}$ transversal.

$-S_G(\sigma) =$ logarithmic modification of $Y \times Y$

intersection product $(\Gamma_\sigma, \Delta_Y)$

graph of σ diagonal

log ~~prod~~ product

$$\underset{\text{smooth}}{Y} \supset \underset{\text{div. w SN.C}}{D = \bigcup_{i=1}^{n} D_i}$$

$$\begin{tikzcd}
\underline{Y} \times Y & Y * Y \arrow[l] \\
Y \arrow[u, "\Delta_Y"] \arrow[ur, "\Delta_Y^{\log}"'] &
\end{tikzcd}$$

blow up at $\underline{D_i \times D_i}$
$i = 1, \ldots, n$

remove proper transform of $D \times Y \cup Y \times D$

logarithmic diagonal.

Example

$$A^d = \mathrm{Spec}\, k[T_1, \dots T_d] \supset D = (T_1 \cdots T_n)$$

$$\bigcup_{i=1}^{n} D_i \quad (= (T_i))$$

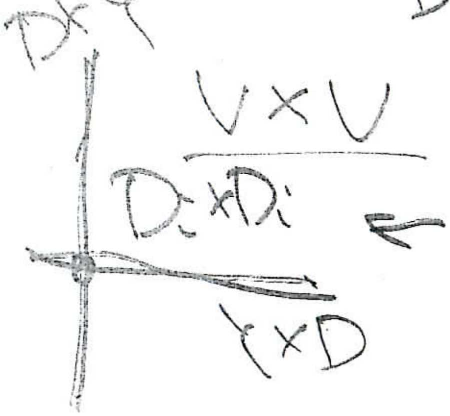
$\Delta^{\log}$ $\quad$ $A^d \xrightarrow{\Delta}$ $\quad \underline{\underline{T_i = S_i}}$

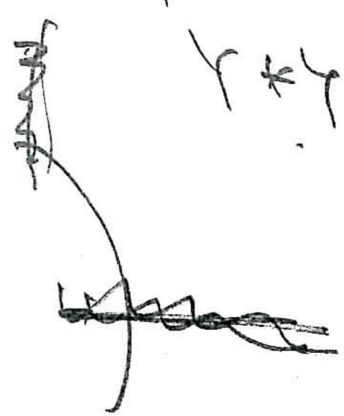
$$A^d \times A^d = \mathrm{Spec}\, \underbrace{k[T_1, \dots, T_d, S_1, \dots, S_d]}_{} = A$$

$U_i = 1$

$$A^d + A^d = \mathrm{Spec}\, A[U_1^{\pm}, \dots ; U_n^{\pm}]$$

$Y \times Y$ $\quad$ D $\quad$ $Y \times Y$





$$(T_i - U_i S_i)$$

$$U_i = \left\{ \frac{T_i}{S_i} \right\}, \quad i = 1, \dots n$$

$$V = Y \setminus D$$

smooth

$$\begin{array}{ccc} \overline{\cancel{Y * Y}} & \hookleftarrow & V \times V & G \curvearrowright V \\ \Delta_Y^{\log} \uparrow & \cup & \cup \quad \cup & \sigma \neq 1 \\ Y & \overline{\Gamma}_\sigma & \hookleftarrow \Gamma_\sigma - \text{graph} & \cap \Delta_V = \phi \end{array}$$

$$(\overline{\Gamma}_\sigma, \Delta_Y^{\log})_{Y * Y} \in CH_0(\underline{Y \setminus V})$$

$$d \qquad Y \qquad 2d \qquad \Delta_Y^{\log}$$

$$S_G(\sigma) \underset{\text{def}}{=} -(\overline{\Gamma}_\sigma, \Delta_Y^{\log})_{Y * Y}$$

$\sigma = 1$ $\quad \delta_G(1)$ by

$$\sum_{\sigma \in G} \delta_G(\sigma) = 0$$

$\delta_G(\sigma) = 0$ unless order of σ

= power of p

log modification $\Rightarrow$ mild ramification

$\dim V = 1$ $\sigma \neq 1$

$$S_G(\sigma) = \sum_{\substack{y \in Y \setminus V \\ \sigma \in I_y}} \underline{\operatorname{ord}_y\left(\frac{\sigma(\pi_y)}{\pi_y} - 1\right) \cdot [y]}$$

$$CH_0(Y \setminus V) = \bigoplus_{y \in Y \setminus V} \mathbb{Z} \cdot y$$

Lefschetz trace formula for open variety

$\in G$

$\sigma \neq 1$

$$\sum_{q=0}^{2\dim V} (-1)^q \operatorname{Tr}(\sigma : H_c^{q}(V_{\bar{k}}, \mathbb{Q}_\ell)))$$

$$= -\underline{\deg}\ \underline{S_G(\sigma)}$$

$\deg : CH_0(Y \setminus V) \longrightarrow \mathbb{Z}$

$\quad\ \ \rotatebox{90}{$\in$} \qquad\qquad\qquad\quad \rotatebox{90}{$\in$}$

$\sum_y n_y [y] \longmapsto \sum_y n_y \deg \cdot y$

If $V = Y$ $(D = \phi)$ usual LTF.

Definition of Swan class

$$\begin{array}{ccc} \text{sch} & & \\ V & \hookrightarrow & Y \quad \mathcal{F} \text{ smooth } \mathbb{Q}_\ell\text{-sheaf on } U \\ G \downarrow \pi & & \bar{\pi} \downarrow \text{ simplifying ass'n} \\ U & \hookrightarrow & X \end{array}$$

$\pi^* \mathcal{F}$ is constant

$$\underset{\substack{\cap \\ CH_0(X \setminus U)}}{Sw_U \mathcal{F}} = \frac{1}{|G|} \sum \underset{\substack{\cap \\ CH_0(Y \setminus V)}}{\bar{\pi}_* S_G(\sigma)} \, \mathrm{Tr}(\sigma : V)$$

$\frac{1}{|G|} \in \mathbb{Q}(\zeta_{p^\infty})$

$\mathrm{Tr}(\sigma : V) \in \mathbb{Z}[\zeta_{p^\infty}]$

negl.

V: rep'n of G

$\bar{\pi}_* S_G(\sigma) = 0$ unless ord σ = power of p

Thm (Kato - S)

U smooth $/k$ $\mathcal{F}$ smooth l-adic sheaf U $\circledR$

$$\chi_c(U_{\bar{k}}, \mathcal{F}) - \mathrm{rk}\,\mathcal{F} \cdot \chi_c(U_{\bar{k}}, \mathbb{Q}_l)$$

$$= -\deg Sw_U(\mathcal{F})$$

traditional method in ramification theory

- kill ramification by ramified covering
- lower numbering filtration
- Swan class to compute Euler number

new method

- kill (partially) ramification
  by blow-up
- upper numbering fil.
- characteristic cycle / class.

2. Ramification groups of local field with non-perfect residue field

C/k curve perfect

$\hookrightarrow$ res. field perfect

X/k variety $\dim > 1$ $\dim D \geq 1$

$\cup$ irred

D divisor $K = \mathrm{Frac}(\widehat{\mathcal{O}_{X,\xi}})$

res fd = Fun fd of D gen pt of D

K complete discrete valuation field

F res fd. not necessary perfect

L/K finite Galois ext'n

$G = Gal(L/K)$

G has two filtrations by ramification gps

— lower numbering $(\underline{G_i})$ $i \in \mathbb{N}$

— $\underline{\underline{\text{upper}}}$ " (G^v) $v \in \mathbb{Q}$, $v > 0$

2-(3

$$G_{i,\text{lvl}} = \ker\left(G \to \underline{\underline{\mathrm{Aut}}}\left(L^\times / 1+m_L^i\right)\right)$$

$$\left(G_i = \qquad \to \mathrm{Aut}\left(\mathcal{O}_L / m_L^i\right)\right)$$

rigid

geometric interpretation

$$\mathcal{O}_L = \mathcal{O}_K[X_1, \ldots X_n] / (f_1, \ldots, f_n)$$

$$\downarrow$$

$$\bar{K}$$

$$G = \mathrm{Hom}_{\mathcal{O}_K\text{-alg}}(\mathcal{O}_L, \bar{K})$$

rigid analytic polydisk

normalized valuation

$$\underline{\underline{G}} = \underline{\underline{f^{-1}(0)}} \subset \underline{\underline{D^n}} = \left\{ \underline{(x_1, \dots x_n)} \in \bar{K} \;\middle|\; v(x_i) \geq 0 \right\}$$

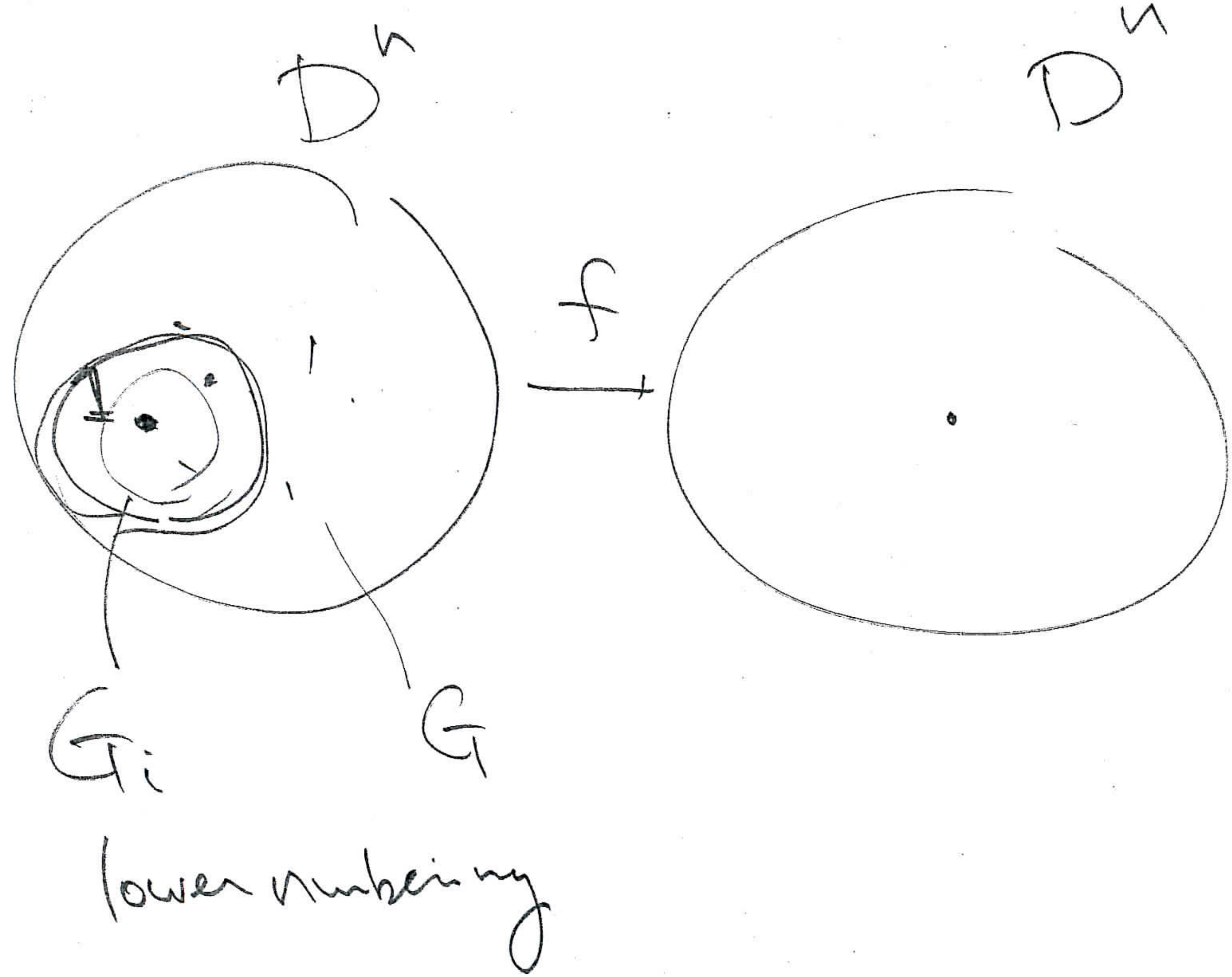
$$\begin{array}{ccc} \boxed{\begin{array}{ccc} f^{-1}(0) & \subset & D^n \\ \downarrow & & \downarrow f \\ 0 & \in & D^n \end{array}} & & \begin{array}{c} (x_1, \dots x_n) \\ \downarrow \\ (f_1(x_1, \dots, x_n), \dots, f_n(x_1, \dots, x_n)) \end{array} \end{array}$$

$$G_i = G \cap D(i, 1) \leftarrow \text{small polydisc}$$

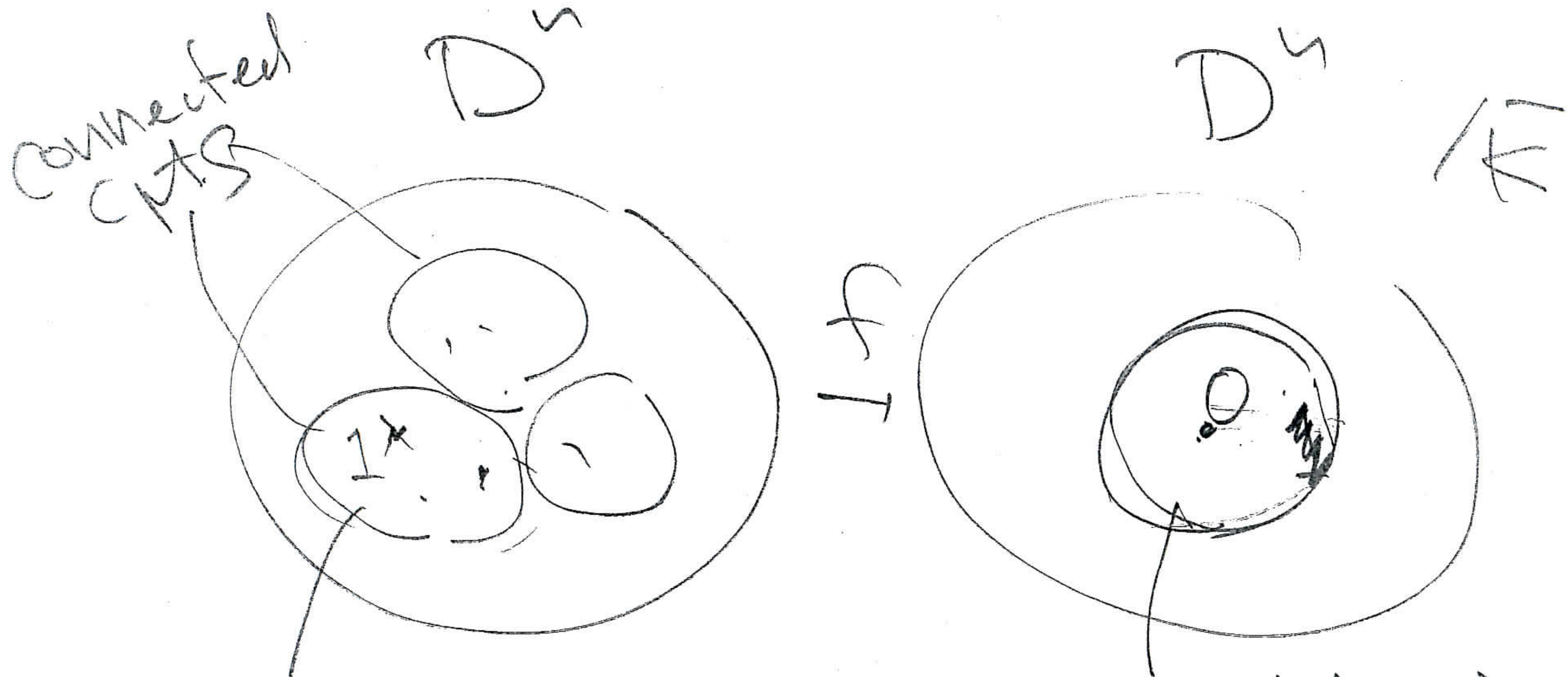
radius   center

$$= \{ \sigma \in G \mid d(\sigma, 1) \leq \| \pi_L^i \| \}$$

uniformizer


D^n
D^n
f
Gi
G
lower numbering

upper numbering filtration



$G^r = \{\sigma \in G \mid$ σ is in the same conn cpt. as the identity $\}$

$\{x \in D^n \mid d(x,0) \leq \|\pi\|^r\}$

unif of K

rigid geometry vs algebraic geom. 2-17

shrinking the radius

blow-up

2-18
3-0

L/K finite Galois ext'n. $G = \mathrm{Gal}(L/K)$

geometric origin

Assume X smooth / k perfect.

$D \subset X$ smooth irred div. $\mathcal{O}_K$

$\xi \in D$ gen. pt $K = \mathrm{Frac}(\widehat{\mathcal{O}}_{X,\xi})$

$$\begin{tikzcd}
E \arrow[d] & \subset & Y \arrow[d] \arrow[dr, phantom, "\square"] & V \arrow[l] \arrow[d, "\substack{\text{finite étale} \\ \text{Galois}}"] & & \mathrm{Spec}\, L \arrow[ll] \arrow[d] \\
D & \subset & X & U = X \setminus D \arrow[l] & & \mathrm{Spec}\, K \arrow[ll]
\end{tikzcd}$$

$G = \mathrm{Gal}(V/U)$