

The eigenvariety for  $OC$  modular symbols

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①

(Pollack-Stevens 5)

(Hecke)

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Eigenpackets

$K/\mathbb{Q}_p$  finite

$\underline{\Lambda}$  = a noeth.  $K$ -algebra ;  $\underline{\Lambda} = A(\Omega)$ ,  $\Omega \subseteq \mathbb{W}$

$\mathcal{R}$  = a commut.  $\underline{\Lambda}$ -algebra ( $\mathcal{R} = \underline{\Lambda}[T_n]$ )

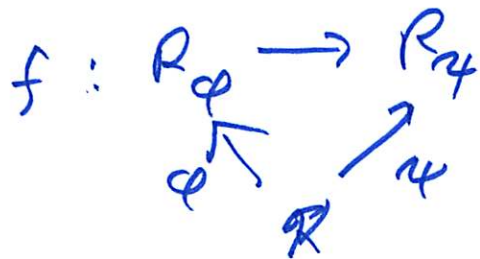
Defn: An eigenpacket is a pair  $(\varphi, R_\varphi)$  ;

~~$R_\varphi$~~   $R_\varphi = K$ -algebra

$\varphi: \mathcal{R} \rightarrow R_\varphi$

The weight  $(\varphi) = \ker(\varphi)$

A morphism  $f: \varphi \rightarrow \psi$  is a homom



If  $H$  is  $R$ -module

(3)

$$\text{let } \mathcal{R}_H = \text{Im}(\mathcal{R} \rightarrow \text{End}(H))$$

$$\varphi_H: \mathcal{R} \rightarrow \mathcal{R}_H$$

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✦ If  $\varphi$  is an eigenpacket

we say  $\varphi$  occurs in  $H$  if

$$\exists \text{ morph } \varphi_H \rightarrow \varphi$$

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Prop: Suppose  $M$  is a f.g.  $R$ -module

(4)

+  $I \subseteq R_M$  ideal

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Let  $\psi$  be any eigenpacket  
+ suppose  $\psi$  reduced

Then  $\psi$  occurs in  $M/IM$

$\iff$  (1)  $\psi$  occurs in  $M$

+ (2)  ~~$\psi$  occurs~~  $\psi(I) = 0$

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$$\begin{array}{l} \omega \in \Omega \cong \mathcal{W} \\ \underline{k \in \mathcal{W}} \end{array}$$

(5)

$$\mathcal{D} \longrightarrow \mathcal{D}_\Omega \longrightarrow \mathcal{D}_k$$

Defn: Let  $\mathfrak{F}$  be a  $k$ -valued eigenpacket

$$\mathcal{O}(k) = \left\{ \mathfrak{F} \text{ eigenpackets, } k\text{-valued} \mid \begin{array}{l} \mathfrak{F} \text{ occurs} \\ \text{in} \\ H_c^1(\mathcal{D}_k) \\ k = w(\mathfrak{F}) \end{array} \right.$$

$H_c^1(\mathcal{D}_k)$  is infinite-dimensional

Why are there eig. vals?



$U_p \in \mathcal{R} =$  Hecke algebra

(6)

$$H_k = H'_c(\mathcal{D}_k) \xrightarrow{U_p} H'_c(\mathcal{D}_k)$$

is completely continuous

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$\Rightarrow$  we can define

$$P_k(T) = \text{Fred}(U_p, H_k) = \det(\mathbb{I} - U_p \cdot T)_{H_k}$$

$P_k(T) \in \mathbb{K}\{\{T\}\}$  entire power series

$$P_k = \prod_{n=1}^{\infty} (1 - \alpha_n(k)T),$$

each  $\alpha_n$  is an eig. value  
of  $U_p$

$$\lim_{n \rightarrow \infty} \alpha_n = 0$$

In fact: for any  $\Omega_0 \subseteq \mathcal{W}$  (7)

$$P_{\Omega_0}(T) = \det(I - T \cdot U_p |_{H_{\Omega_0}}) \in A(\Omega_0) \{T\}$$

$$H_{\Omega_0} = H'_c(\mathcal{D}_{\Omega_0})$$

$$P_{\Omega_0} = 1 + \sum_{n=1}^{\infty} a_n T^n,$$

$$a_n \in (A(\Omega_0))^0$$

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$$\Omega_0 \subseteq \Omega_1$$

$$P_{\Omega_1} |_{\Omega_0} = P_{\Omega_0}$$

$$\Rightarrow a_n \in \Lambda.$$

$$\llcorner \mathbb{Z}_p[[\mathbb{Z}_p^*]]$$





(b)  $H_Q, H'$  are invariant for  $\overline{U}_P$  (9)

$$\text{Fred}(H_Q) \cdot \text{Fred}(H') = \text{Fred}(H_{\Omega_0})$$

"  $\downarrow$   
 $\mathbb{P}_{\Omega_0}$

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$H_Q$  fin. gen. as  $A$ -module.

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Define:  $\mathcal{R}(Q) =$  image of Hecke in  $\text{End}(H_Q)$

$\Rightarrow \mathcal{R}(Q)$  is an affinoid algebra

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$$\mathcal{R} \rightarrow \mathcal{R}(Q)$$

Let  $\overline{\mathcal{X}}(Q) =$  affinoid associated to  $\mathcal{R}(Q)$

Final Fact:

(6)

$$\forall k \in \Omega_0$$
$$H_Q \otimes_{A, k} K = H_c^1(\mathcal{O}_k) \left( \begin{matrix} * \\ \neq 0 \end{matrix} \right)$$
$$\cup$$

$$H_Q / I_k H_Q.$$

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View  $\overline{X}(Q)_k \cong \mathcal{O}(k)$