The eigenvariety for OC modular symbols

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(Pollack-Stevens 5)

(Hecke)

Eigenpackets

$K/\mathbb{Q}_p$ finite

Λ = a noeth. K-algebra ; $\Lambda = A(\Omega)$, $\Omega \subseteq \mathcal{W}$

$\mathcal{R}$ = a commut. Λ-algebra $\left(\mathcal{R} = \Lambda[T_n] \right)$

Defn: An eigenpacket is a pair

(φ, R_φ) ;

$R_\varphi = K$-algebra

$\varphi : \mathcal{R} \twoheadrightarrow R_\varphi$

The weight $(\varphi) = \ker(\varphi)$

A morphism $f : \varphi \longrightarrow \psi$ is a homom

$$\begin{tikzcd} f : R_\varphi \arrow[rr] & & R_\psi \\ & \mathcal{R} \arrow[ul, "\varphi"] \arrow[ur, "\psi"'] & \end{tikzcd}$$

If H is $\mathcal{R}$-module

let $\mathcal{R}_H = \mathrm{Im}(\mathcal{R} \to \underset{\Lambda}{\mathrm{End}}(H))$

$\varphi_H : \mathcal{R} \to \mathcal{R}_H$

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* If φ is an eigenpacket we say φ occurs in H if $\exists$ morph $\varphi_H \to \varphi$

<u>Prop:</u> Suppose M is a f.g. $\mathcal{R}$-module

& $I \subseteq \mathcal{R}_M$ ideal.

Let ψ be any eigenpacket

& suppose ψ reduced

Then ψ occurs in M/IM

$\Longleftrightarrow$ (1) ψ occurs in M

& (2) ~~ψ occurs~~ $\psi(I) = 0$

$\Omega \subseteq W$

$k \in W$

(5)

$$\mathcal{D} \longrightarrow \mathcal{D}_\Omega \longrightarrow \mathcal{D}_k$$

<u>Defn:</u> Let ξ be a K-valued eigenpacket

$$\mathcal{E}(k) = \left\{ \xi \text{ eigenpackets, K-valued} \;\middle|\; \begin{array}{l} \xi \text{ occurs in} \\ H^1_c(\mathcal{D}_k) \\ k = w(\xi) \end{array} \right.$$

$H^1_c(\mathcal{D}_k)$ is infinite-diml

Why are there eig. vecs?

$U_p \in \mathcal{R}$ = Hecke algebra

$$H_k = H^1_c(\mathcal{D}_k) \xrightarrow{U_p} H^1_c(\mathcal{D}_k)$$

is completely continuous

$\Longrightarrow$ we can define

$$P_k(T) = \mathrm{Fred}(U_p, H_k) = \det(I - U_p \cdot T \mid H_k)$$

$$P_k(T) \in K\{\{T\}\} \quad \text{entire power series}$$

$$P_k = \prod_{n=1}^{\infty} (1 - \alpha_n(k) T),$$

each α_n is an eig. value of U_p

$$\lim_{n \to \infty} \alpha_n = 0$$

In fact: for any $\Omega_0 \subseteq \mathcal{W}$ (7)

$$P_{\Omega_0}(T) = \det\left(I - T \cdot U_p \mid H_{\Omega_0}\right) \in A(\Omega_0)\{\{T\}\}$$

$$H_{\Omega_0} = H^1_c(\mathcal{D}_{\Omega_0})$$

$$P_{\Omega_0} = 1 + \sum_{n=1}^{\infty} a_n T^n,$$

$$a_n \in \left(A(\Omega_0)\right)^0$$

$\Omega_0 \subseteq \Omega_1$

$P_{\Omega_1}\big|_{\Omega_0} = P_{\Omega_0}$ $\Rightarrow$ $a_n \in \Lambda.$

$\Lambda = \mathbb{Z}_p[[\mathbb{Z}_p^*]]$

<u>Suppose</u>

$$f_{\Omega_0} = Q \cdot F$$

where Q is a fredholm polynom

F is a fredholm series

$\&$ $(F, Q) = 1$

<u>Then</u> Coleman $\Longrightarrow$

$$H_{\Omega_0} = H_Q \oplus H'$$

where H_Q, H' are ~~free~~ Ω/-able $A(\Omega_0)$-modules $\overset{\shortparallel}{\Lambda}$

$\&$ ⓐ $\mathrm{rank}_{A(\Omega_0)} H_Q = \deg(Q)$

(b) H_Q, H' are invariant for U_p

$$\mathrm{Fred}(H_Q)\cdot \mathrm{Fred}(H') = \mathrm{Fred}(H|_{\Omega_0}) = P_{\Omega_0}$$

H_Q fin. gen. as Λ-module.

<u>Define</u>: $\mathcal{R}(Q)$ = image of Hecke in $\mathrm{End}(H_Q)$

$\Longrightarrow$ $\mathcal{R}(Q)$ is an affinoid algebra

$$\mathcal{R} \twoheadrightarrow \mathcal{R}(Q)$$

Let $X(Q)$ = affinoid associated to $\mathcal{R}(Q)$

Final fact:

$\forall k \in \Omega_0$

$$H_Q \otimes_{\Lambda, k} K = H_c^1\left(\mathcal{D}_k^{(Q \stackrel{*}{=} 0)}\right)$$

$$\parallel$$

$$H_Q / I_k H_Q.$$

View $\mathbb{X}(Q)_k \subseteq \mathcal{C}(K)$