

Quadratic forms and Automorphic forms (AWS '09 outline)

Course Outline:

1. Local/Global Invariants
 - a. Local Rational Invariants
 - i. Dim, Det, Hasse invariant
 - ii. Isotropic/anisotropic dimensions, signature
 - b. Local Integral Invariants
 - i. Jordan Decompositions
 - ii. Local Densities
 - iii. Difficulties when $p=2$
 - c. Global Invariants
 - i. Class number
 - ii. Automorphisms
 - iii. Representation numbers and Theta series

2. Transformation formulas for Theta series
 - a. Fourier transforms and Gaussians
 - b. Poisson Summation
 - c. Computation of Gauss sums
 - d. Modular Forms
 - e. Circle Method and Siegel's Formula (Siegel-Weil Formula?)
 - f. The Basis Problem

3. Clifford Algebras and Quaternion Algebras
 - a. The Clifford Algebra
 - i. Explicitly when $n \leq 4$
 - b. Quaternion Algebras
 - i. Local/rational invariant theory
 - ii. Orders and Eichler Orders
 - iii. Class numbers and Eichler's Trace formula
 - iv. Relation to quadratic subfields
 - v. Norm Forms and Quadratic forms
 - c. Arithmetic Representation results for Ternary quadratic forms
 - d. Results of Shimura for existence of genera over number fields

4. Automorphic Forms
 - a. Algebraic Modular Forms vs. Classical Modular Forms
 - b. Gross Curves
 - c. The Weil representation
 - d. Theta series as a lift from the quaternion algebra
 - e. Siegel's theorem revisited
 - i. Representation theory
 - ii. Eisenstein series -- Euler product
 - f. Hecke Actions??

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Possible Project Ideas:

1. Explicit representation results via quaternion algebras (e.g. Donkar's Thesis)
2. Computing class numbers of quaternion algebra and quadratic forms
3. Trace formula computations for Eichler-Deuring Mass formula
4. Explicit Basis problem computations for certain spaces of modular forms
5. Explicit representation formulas from Siegel's theorem for certain forms
6. Compatibility of Hecke Actions through the Weil rep'n
7. Computation of Models for the Weil representation
8. Generalizations for Hermitian forms.