Octics:

$\mathbb{P}^4_{(1,1,2,2,2)}[8]$:

$$(x_1, x_2, x_3, x_4, x_5) \sim (\lambda x_1, \lambda x_2, \lambda^2 x_3, \lambda^2 x_4, \lambda^2 x_5) \sim (-x_1, -x_2, x_3, x_4, x_5)$$

so there is a $\mathbb{Z}_2$ action with fixed point set $\{(0, 0, x_3, x_4, x_5)\}$

$$\mathcal{M} : P = x_1^8 + x_2^8 + x_3^4 + x_4^4 + x_5^4 - 2\phi x_1^4 x_2^4 - 8\psi x_1 x_2 x_3 x_4 x_5$$

$$G : \quad (x_1, x)2, x_3, x_4, x_5) \to (\alpha^{n_1} x_1, \ldots, \alpha^{n_5} x_5)$$

$$\text{where } \alpha^8 = 1, \quad a = n_1 + n_2 + 2n_3 + 2n_4 + 2n_5 \equiv 0 \pmod 8$$

$$G \cong \mathbb{Z}_4^3, \qquad \text{The mirror is } \mathcal{W} = \widehat{\mathcal{M}/G}$$

$$h^{1,1}(\mathcal{M}) = 2, \quad h^{2,1}(\mathcal{M}) = 86 = 83 + 3$$

We can define another action of G:

$$(x_1, \ldots, x_5, \psi, \phi) \to (\alpha^{n_1} x_1, \ldots, \alpha^{n_5} x_5, \psi\alpha^{-a}, \phi\alpha^{-4a})$$

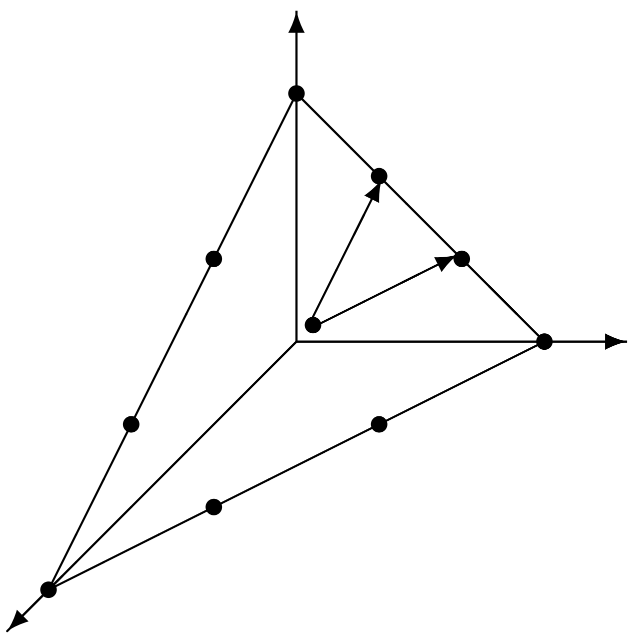
so the natural coordinates are $\psi^8, \psi^4\phi, \phi^2$.

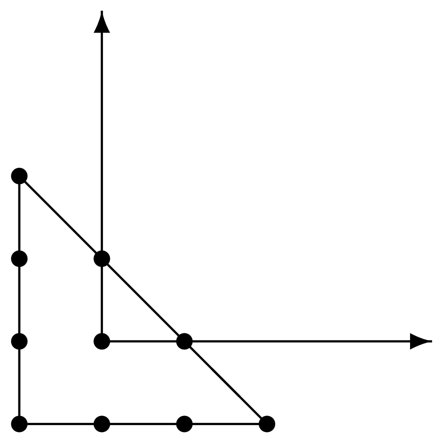
We have the polyhedra Δ, ∇.

$$h^{2,1} = \mathrm{pts}(\Delta) - \sum_{\substack{\mathrm{codim}\,\theta=1 \\ \theta\in\Delta}} \mathrm{int}(\theta) + \sum_{\substack{\mathrm{codim}\,\theta=2 \\ \theta\in\Delta}} \mathrm{int}(\theta)\mathrm{int}(\theta^\star) - 5$$

$$h^{1,1} = \mathrm{pts}(\nabla) - \sum_{\substack{\mathrm{codim}\,\theta^\star=1 \\ \theta^\star\in\nabla}} \mathrm{int}(\theta^\star) + \sum_{\substack{\mathrm{codim}\,\theta^\star=2 \\ \theta^\star\in\nabla}} \mathrm{int}(\theta^\star)\mathrm{int}(\theta) - 5$$

$$\sum_{i=1}^{3} x_i^3 - 3\psi x_1 x_2 x_3 = 0$$





$$P = c_1 y_1^8 + c_2 y_2^8 + c_3 y_3^4 + \ldots + c_6 y_1^4 y_2^4 + c_7 y_1 y_2 y_3 y_4 y_5$$

D_0	1	1	1	1	1	1
D_1	1	8	0	0	0	0
D_2	1	0	8	0	0	0
D_3	1	0	0	4	0	0
D_4	1	0	0	0	4	0
D_5	1	0	0	0	0	4
D_6	1	4	4	0	0	0

$$D_0 = -(D_1 + \ldots + D_6)$$

$$D_0 = -4D_3 = -4D_4 = -4D_5$$

$$D_0 = -8D_1 - 4D_6$$

$$D_0 = -8D_2 - 4D_6$$

$$D_3 = D_4 = D_5 = H$$

$$D_1 = D_2 = L$$

$$D_6 = H - 2L$$

$$H^3 = 8, \quad H^2 L = 4, \quad H L^2 = L^3 = 0$$

Generators of the Mori cone:

$$\mathbf{h} = (-4, 0, 0, 1, 1, 1, 1) \qquad \mathbf{l} = (0, 1, 1, 0, 0, 0, -2)$$

$$\mathbf{h} \cdot H = 1, \quad \mathbf{h} \cdot L = 0$$
$$\mathbf{l} \cdot H = 0, \quad \mathbf{l} \cdot L = 1$$

$$\lambda = c^{\mathbf{h}} = \frac{c_3 c_4 c_5 c_6}{c_0^4} = \frac{-2\phi}{(8\psi)^4}$$

$$\mu = c^{\mathbf{l}} = \frac{c_1 c_2}{c_6^2} = \frac{1}{(2\phi)^2}$$

$$P = x_1^8 + x_2^8 + x_3^4 + x_4^4 + x_5^4 - 2\phi x_1^4 x_2^4 - 8\psi x_1 \ldots x_5$$

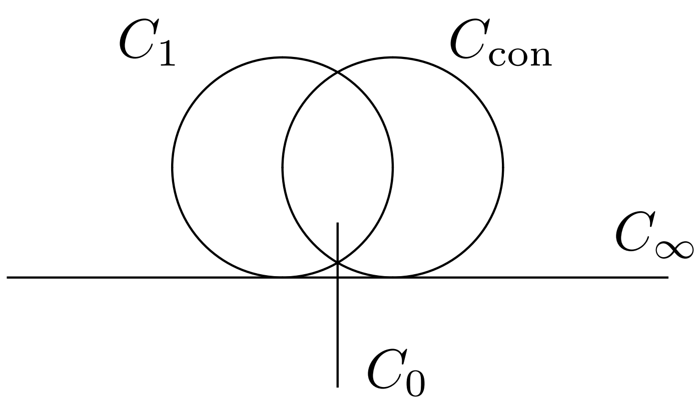
For which ϕ, ψ is the manifold singular?

C_{con}: $(\phi + 8\psi^4)^2 - 1 = 0$, conifold locus: $\mathcal{M}$ has nodes.

C_1: $\phi^2 = 1$, $\mathcal{M}$ has 4 rather singular points

C_∞: $\phi, \psi \to \infty$, $\mathcal{M}$ singular

C_0: $\psi = 0$, orbitfold $\psi \simeq \alpha\psi$



Counting the number of points:

$$\sum_{y \in \mathbb{F}_p} \Theta(yP) = \delta(P(x))$$

$$\Theta(yP) = \Theta(-8\psi y x_1 x_2 \ldots x_5)\Theta(-2\phi y x_1^4 x_2^4)\Theta(y x_1^8)\Theta(y x_2^8)\Theta(y x_3^4)\Theta(y x_4^4)\Theta(y x_5^4)$$

$$\Theta(\xi) = \frac{1}{p-1} \sum_{m=0}^{p-2} G_{-m} \mathrm{Teich}(\xi)^m$$

allows to evaluate everything in terms of Gauss sums

Zeta function:

$$\mathcal{M} : \zeta(T,\psi,\phi) = \frac{R_1 \prod_{\mathbf{v}} R_{\mathbf{v}}^{\gamma_{\mathbf{v}}}}{(1-T)(1-pT)^2(1-p^2T)^2(1-p^3T)}$$

where R_1 is a sextic and

$$R_1(\frac{1}{p^3T}) = \frac{1}{p^9T^6}R_1(T)$$

$$\mathcal{W} : \zeta_{\mathcal{W}} = \frac{R_1}{(1-T)(1-pT)^{83}(1-p^2T)^{83}(1-(\frac{\phi^2-1}{p})pT)^3(1-(\frac{\phi^2-1}{p})p^2T)^3(1-p^3T)}$$