$\mathbb{P}^4[5]$

$$h^{p,q} = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 1 & & 0 & \\ 1 & & 101 & & 101 & & 1 \\ & 0 & & 1 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

The mirror image is

$$\mathcal{W} = \widehat{\mathcal{M}/G}$$

Kähler class:

$$h^{1,1}(\mathcal{W}) = h^{2,1}(\mathcal{M}) = 101$$

$$\zeta_{\mathcal{W}} = \frac{R_0}{(1-T)(1-pT)^{101}(1-p^2T)^{101}(1-p^3T)} =$$

$$= \frac{R_0}{(1-T)(1-pT)(1-p^2T)(1-p^3T)} \cdot \frac{1}{(1-pT)^{100}(1-p^2T)^{100}}$$

Fundamental period:

$$\varpi(\lambda,\epsilon) = \sum_{n=0}^{\infty} A_n(\epsilon)\lambda^{n+\epsilon} = \sum_{k=0}^{3} \frac{1}{k!}\varpi_k(\lambda)$$

$$\mathcal{L}\varpi = 0 \text{ has 4 solutions } \varpi_0, \varpi_1, \varpi_2, \varpi_3; \qquad \epsilon^4 = 0,\ \epsilon^3 \neq 0$$

We can calculate $N = \frac{\nu-1}{p-1}$ instead, in terms of $\varpi_0, \varpi_1, \varpi_2, \varpi_3$

We can set $p^4 = 0$

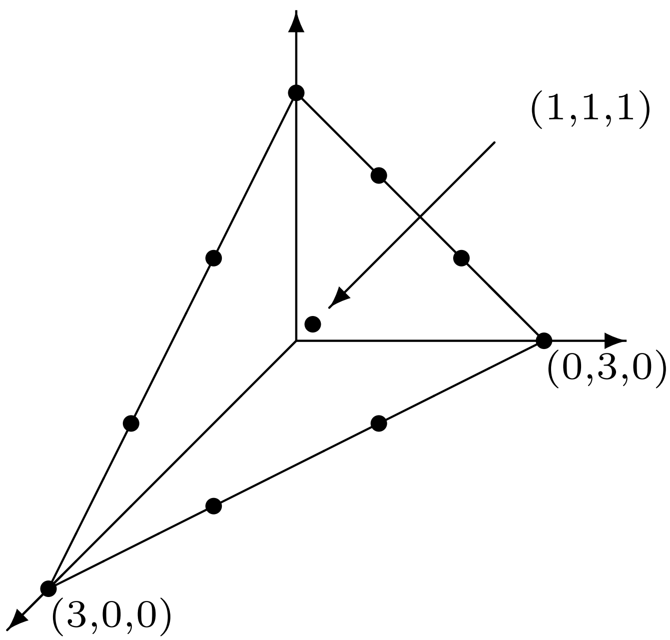
$$\epsilon^4 = 0, \qquad H^4 = 0$$

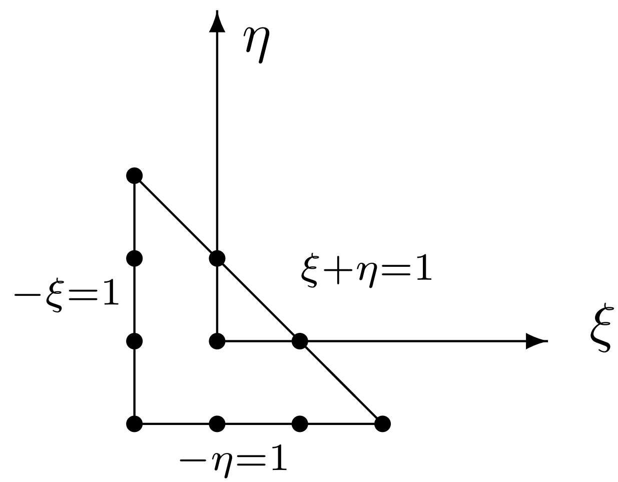
For the quartic, $h^{1,1} = 1$

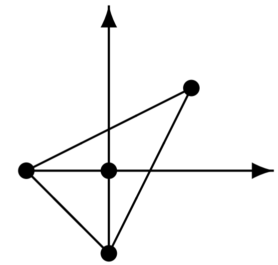
For the octic, $h^{1,1} = 2$

$$\mathbb{P}^2[3]: \qquad \sum_{i=1}^{3} x_i^3 - 3\psi x_1 x_2 x_3 = 0$$

$$x^{\mathbf{m}}, \quad \mathbf{m} = (m_1, m_2, m_3), \quad m_i \geq 0, \quad m_1 + m_2 + m_3 = 3$$







We get a pair of reflexive polyhedra:

$$\Delta = \text{Newton polyhedron}$$

$$\nabla = \text{polyhedron over the fan of } \mathbb{P}^2$$

$$\mathcal{M} \simeq (\Delta, \nabla)$$

Conversely, the toric data constructs a family of manifolds containing $\mathcal{M}$.

$$\mathcal{W} \simeq (\nabla, \Delta)$$

A point in Δ is a monomial corresponding to $\mathcal{M}$ and a divisor of the toric variety in which $\mathcal{W}$ is a hypersurface.

$\mathcal{M}$ a CY hypersurface in $\mathbb{P}_\nabla$

$$P(x) = \sum_{\substack{\mathbf{m}\in\Delta \\ \mathbf{m}\neq(1,1,1,1,1)}} c_\mathbf{m} x^\mathbf{m} - c_0 Q$$

$$\varpi(c,D) = \frac{\prod'_\mathbf{m} \Gamma(D_\mathbf{m}+1)}{\Gamma(-D_0+1)} \sum_{\gamma\in V_\nabla} \frac{\Gamma(-\gamma D_0 - D_0 + 1)}{\prod'_\mathbf{m} \Gamma(\gamma D_m + D_m + 1)} c^{\gamma+D}$$

Where $D_\mathbf{m}$ are the toric divisors of $\mathbb{P}_\nabla$,

$$c^{\gamma+D} = \prod_{\mathbf{m}\in\Delta} c_\mathbf{m}^{\gamma_\mathbf{m}+D_\mathbf{m}}, \qquad D_0 = -\sum_\mathbf{m}{}' D_\mathbf{m}$$

and the sum is over γ in the Mori cone of $\mathbb{P}_\nabla$.

Apply to the mirror quintic:

$$\begin{array}{c} D_0 \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{array} \quad \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 5 & 0 & 0 & 0 & 0 \\ 1 & 0 & 5 & 0 & 0 & 0 \\ 1 & 0 & 0 & 5 & 0 & 0 \\ 1 & 0 & 0 & 0 & 5 & 0 \\ 1 & 0 & 0 & 0 & 0 & 5 \end{pmatrix}$$

The columns give relations between the divisors:

$$D = (D_0, D_m), \quad D_0 = -\sum{}' D_m, \quad \gamma = (\gamma_0, \gamma_m), \quad \gamma_0 = -\sum{}' \gamma_m$$

$$D_0 = -5D_1$$

$$D_0 = -5D_2$$

$$\vdots$$

$$D_0 = -5D_5$$

$$D_1 = D_2 = \ldots = D_5 = H$$

$$D_0 = -5H$$

$$P(x) = c_1 x_1^5 + c_2 x_2^5 + \ldots + c_5 x_5^5 - c_0 x_1 x_2 x_3 x_4 x_5$$

Change of coordinates: $c_i^{-1/5} y_i = x_i$:

$$P(y) = y_1^5 + y_2^5 + \ldots + y_5^5 - 5\psi y_1 y_2 y_3 y_4 y_5$$

$$\text{where } 5\psi = \frac{c_0}{(c_1 c_2 c_3 c_4 c_5)^{1/5}}$$

$$\text{or } \lambda = \frac{1}{(5\psi)^5} = \frac{c_1 c_2 c_3 c_4 c_5}{c_0^5} = c^{\mathbf{k}}$$

and $\mathbf{k} = (-5, 1, 1, 1, 1, 1)$ generates the Mori cone

The γ in the Mori cone are then $\gamma = n\mathbf{k}$, and

$$\mathbf{k} \cdot H = 1, \; -\gamma D_0 = -\gamma(-5H) = 5n, \text{ so}$$

$$\varpi(\lambda, H) = \frac{\Gamma^5(H+1)}{\Gamma(5H+1)} \sum_{n=0}^{\infty} \frac{\Gamma(5n+5H+1)}{\Gamma^5(n+H+1)} \lambda^{n+H}$$