

Instanton Sums and Monodromy

String theory

10-dim spacetimes

$X^6 \times M^4$
↑
Minkowski

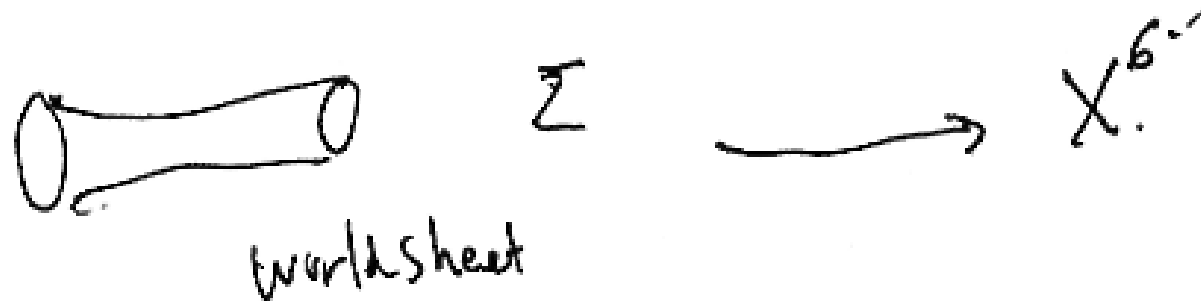
want supersymmetry (SUSY) in 4D.

\Rightarrow covariantly constant spinor on X^6 .

holonomy is restricted: $SU(3)$

\Rightarrow Ricci flat (Calabi),

$\Rightarrow X^6$ is a projective alg. variety.



- Maps (Σ, X^6) .

quantum field theory

Expansion in g , coefficients are
instanton numbers⁹.

want all maps in a fixed coh. class
with minimum action. $\eta_{\vec{d}}$

$$\eta_{\vec{0}} + \sum \eta_{\vec{d}} \mathcal{L}^{\vec{d}} = \text{physical quantity}$$

e.g. $\eta_{\vec{d}} = \text{Gram-Wittin invariants of } X.$

$$X^6 \in \mathbb{C}P^4$$

$$X^6 = \left\{ \begin{array}{l} X_1^5 + X_2^5 + \dots + X_5^5 = 0 \end{array} \right\}$$

QFT assoc. to X^6

can sometimes be described by a

"gauged linear sigma model".

Instantons are easy to calculate.

Related to original QFT by "renormalization".

renormalization \Rightarrow need a parameter change
(in g) when comparing answers.

Superspace

P. Deligne, P. Freed

"Supersolutions" in: Quantum Fields & Strings, vol 1,
ed by Deligne et al, AMS 1998?

A superspace is a supermanifold $\leftarrow \mathbb{Z}_2$ -graded
whose "body" is Minkowski space.

$M^{2|32}$

x^0, x^1 : coords on M^2
 $(x^0)^2 - (x^1)^2$

$\partial_{\pm} = \frac{\partial}{\partial x^0} \pm \frac{\partial}{\partial x^1}$, even vector fields

odd vector fields: $\bar{D}_+, \bar{D}_-, D_+, D_-$

over view

$$[D_+, D_+] = \dots = 0$$

$$[D_-, D_-] = \dots = 0$$

$$[D_+, \bar{D}_+] = -2a_+$$

$$[D_-, \bar{D}_-] = -2a_-$$

\mathcal{L} = Super Lie algebra = extension of Poincaré alg.
by these odd v.f.

$M^{2|2}(2,2)$ is acted upon by $\exp(\mathcal{L})$.

D_{2n}, \mathcal{Q}_+ are left-invariant v.f. on $M^{2|2}(2,2)$

$\Phi : M^{2|2}(2,2) \longrightarrow \mathbb{C}^n$ Super field.

eg. $D_+ \Phi / M^2$, $D_- \Phi / M^2$ this produces
"usual" fields

$$\Phi / M^2 = \varphi : M^2 \rightarrow \mathbb{C}^n.$$

Chiral superfield : $\overline{D}_+ \Phi_i = \overline{D}_- \frac{\Phi_i}{1} = 0.$

(holomorphic)

Need compact abelian group G .

$$\rho : G \rightarrow U(1)^n \subseteq GL(n, \mathbb{C}).$$

we will also need

$$e \in \text{Hom}(g, \mathbb{R})$$

$W_1 = G$ -invariant poly on \mathbb{P}^1 .

$$\mathcal{Z} = \int d^4\theta \left(\int e^{-i d\rho^\theta V} \left\| \frac{1}{\sqrt{e}} \right\|_{\Sigma_1} \right)$$

G -invariant
(kinetic terms)

$$d^4\theta = d\theta^+ d\theta^- d\bar{\theta}^+ d\bar{\theta}^-$$

$V =$ gauge multiplet: $M^{2(2,2)} \rightarrow \mathfrak{g}$

$$\Sigma = \bar{D}_+ D_- V.$$

"twisted chiral"

$$\bar{D}_+ \Sigma = D_- \Sigma = 0.$$

Need norm $\sim \mathfrak{g}$.

$\mathcal{L} = \text{kinetic system}$

$$\vdash \operatorname{Re} \int d\theta^+ d\theta^- W(\Phi_1, \dots, \Phi_n)$$

$$+ \operatorname{Im} \int d\theta^+ d\theta^- \langle \mathcal{L}, \Sigma \rangle$$

$$z \in \operatorname{Hom}(\mathfrak{g}, \mathbb{C})$$

$w = \text{poly.}$

Coefs of $w.$ = params

$\rho: G \rightarrow U(1)^n.$

$z \in \text{Hom}(g, \mathbb{C})$ more params.

Example

$$W(X_0, X_1, \dots, X_5) \\ = X_0 X_1^5 + \dots + X_0 X_5^5 \\ + X_0 X_1 X_2 X_3 X_4 X_5$$

$$G = U(1) \quad \frac{X_0 \quad X_1 \quad \dots \quad X_5}{\begin{matrix} \leftarrow Q_i^a \\ \leftarrow \\ \leftarrow \\ \leftarrow \\ \leftarrow \\ \leftarrow \end{matrix}} \\ (t_i \mapsto (x_0, x_1, \dots))$$

Low-energy analysis

(4)

Pick a basis for \mathfrak{g} .

$$V = (V_1, \dots, V_{n-d}).$$

group action $\mathbb{F}_1 \ni e^{V_a} : \mathbb{F}_1 \rightarrow (e^{V_a})^{\mathbb{F}_1} \mathbb{F}_1$

Derivatives of certain component fields do not occur
in action! can solve:

$$z = ir + \frac{1}{2\pi} \theta.$$

$$D_a = -e^2 \left(\sum Q_i^a |\phi_i|^2 - r_a \right)$$

$$F_i = - \frac{2W}{2\phi_i}$$

↑
Comp of $\bar{\Psi}_i$

Potential for bosonic fields

$$U = \frac{1}{2e^2} \sum D_a^2 + \sum |F_i|^2 + \dots$$

To minimize u , must have

$$D_a = 0, \quad F_i = 0.$$

Example

$$\rightarrow |\varphi_0|^2 + |\varphi_1|^2 + |\varphi_5|^2 - r = 0.$$

$$W(\vec{x}) = x_0 P_5(\vec{x})$$

$$\frac{\partial W}{\partial x_0} = P_5(\vec{x}) = 0$$

$$\frac{\partial W}{\partial x_i} = x_0 \frac{\partial P_5}{\partial x_i}(\vec{x}) = 0$$

if $r < 0$ then not all ϕ_1, \dots, ϕ_5 can
vanish.

$$\mathbb{C} \times \{\mathbb{C}^5 \setminus \{0\}\} \cong \mathbb{C} \times \boxed{S^4 / U(1)}$$

get a complex line bundle $\mathbb{C}P^4$
 $\mathcal{O}(-5)$
over $\mathbb{C}P^4$.

¹
if P is generic,

~~let $\frac{\partial P}{\partial x_i}$~~

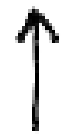
$P, \frac{\partial P}{\partial x_i}$ will share no common zero
outside $\vec{0} \in \mathbb{C}^5$.

$\Rightarrow P=0, X_0=0.$



\mathbb{C}^5

$$\Rightarrow P=0, X_0=0.$$



$$\mathbb{C}P^4$$

$$\text{(quartic hyp)} \subseteq \mathbb{C}P^4$$

(Calabi-Yau).

Constraint on choices

$$p: G \rightarrow U(1)^n, \quad W(x_1, \dots, x_n), \quad G\text{-module}$$

family of theories with coeffs varying

{ moments in W }.

$$X^{t_{\mathbb{R}}} = \prod X_i^{t_{\mathbb{R}i}}$$

t_a = row of a nonnegative integer matrix T .

G ~~has~~ ^{determini} a set of G -invariant Laurent monomials.

P has rows P_{ai} integers.

$$x^{Pa} = \prod x_i^{P_{ai}}$$

$G \subseteq U(n)^m$.

$$T = SP.$$



nonnegative
int

rank d.

Mirror theory:

$$t_T = t_p t_s.$$

↑
new moment.

↑
new gaps

conformally invariant theory in IR.!

$\exists \vec{\mu}, \vec{\nu}$ radial vectors

$$T \vec{\mu} = (1 \ 1 \ 1 \ 1 \rightarrow)^T$$

$$\vec{\nu}^T T = (1 \ 1 \ 1 \ 1 \ \dots \ 1)$$