Instanton Sums and Monodromy

David R. Morrison
Duke University

String theorists are interested in several quantum field theories and string theories associated with Calabi–Yau threefolds. The most straightforward of these to formulate is the two-dimensional quantum field theory which governs the physics of a string propagating on the Calabi–Yau threefold. However, making calculations directly with that quantum field theory is quite difficult. To the extent that they can be made, some amazing things have been discovered about Calabi–Yau threefolds, including the famous counting of rational curves (as "instantons") on the quintic threefold.

In 1992, Witten proposed an alternative way to study these quantum field theories for certain Calabi–Yau threefolds, by means of his "gauged linear sigma model" (GLSM). The connection to the Calabi–Yau threefold is less direct, but the model has some features which make calculations easier in many cases. For example, the instantons in this model can often be calculated explicitly with some ease, although they have a different mathematical interpretation (i.e., they do not directly count rational curves on the Calabi–Yau threefold). When gathered into an appropriate "instanton sum," they are expected to express intrinsic information about the physical model, and so be ultimately related to the famous curve-counting problem. Many remarkable features of Calabi–Yau threefolds and their moduli spaces can be seen from calculation involving these GLSM instantons.

The course will describe the mathematical aspects of these GLSM theories, focusing on instantons and also on monodromy of various structures defined over the moduli space. The basic reference is [1], although that paper was not written with a mathematical audience in mind.

The student project will study these GLSM instanton sums, and other features of the moduli space, explicitly in the case of a particular Calabi–Yau threefold (the so-called (2,86) model). The geometry of the (2,86) model is described many places, such as [2]. Some aspects of the GLSM for this model were discussed in [3].

Useful mathematical orientation is provided in [4], section 3.3 of [5], and [6].

References

[1] D. R. Morrison and M. R. Plesser, *Summing the instantons: Quantum cohomology and mirror symmetry in toric varieties*, Nuclear Phys. B **440** (1995), 279–354, `hep-th/9412236`.

[2] B. R. Greene, D. R. Morrison, and A. Strominger, *Black hole condensation and the unification of string vacua*, Nuclear Phys. B **451** (1995), 109–120, `hep-th/9504145`.

[3] B. R. Greene, D. R. Morrison, and C. Vafa, *A geometric realization of confinement*, Nuclear Phys. B **481** (1996), 513–538, `hep-th/9608039`.

[4] D. R. Morrison, *Making enumerative predictions by means of mirror symmetry*, Mirror Symmetry II (B. Greene and S.-T. Yau, eds.), International Press, Cambridge, 1997, pp. 457–482, `alg-geom/9504013`.

[5] D. R. Morrison, *Through the looking glass*, Mirror Symmetry III (D. H. Phong, L. Vinet, and S.-T. Yau, eds.), AMS/IP Studies in Advanced Mathematics, vol. 10, American Mathematical Society and International Press, 1999, pp. 263–277, `alg-geom/9705028`.

[6] D. R. Morrison, *Geometric aspects of mirror symmetry*, Mathematics Unlimited – 2001 and Beyond (B. Enquist and W. Schmid, eds.), Springer-Verlag, 2001, pp. 899-918, `math.AG/0007090`.