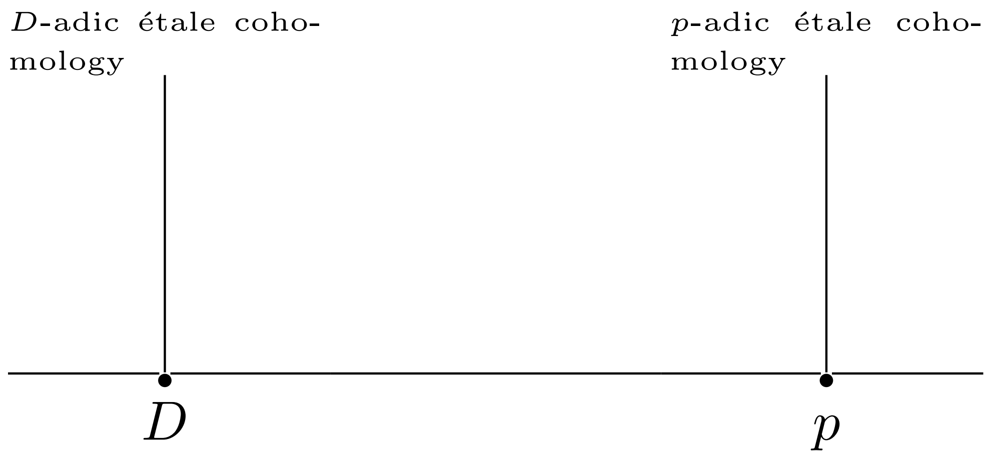
Further Uniformities

1.



I =primes

2. Can analyze maximal unramified extension of $\mathbb{Q}_p$

3. Witt vectors $\mathbf{Witt}(\mathbb{F}_p^{\mathbf{alg}})$ with Witt Frobenius

4. Analytic situations over $\mathbb{Q}_p$

Michael Morley(1965)

- $L:$ Tarski Space(L)= Set of complete theories

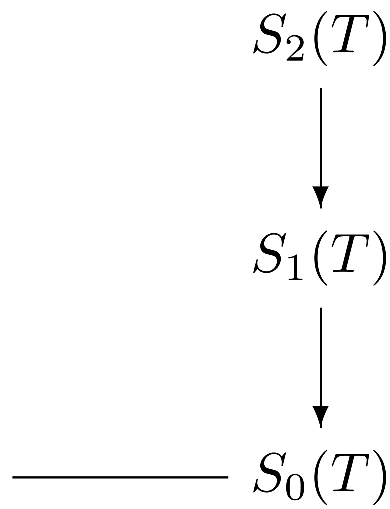
  Basic opens determined by sentences

  ↑ Language

- $S_0(T)$: Closed subset of spaces containing T

$S_1(T) =$ extend by a constant v

$S_0(T)$ in a new language

- 

Tarski

- What happens if we add **exp** as a primitive to $\mathbb{R}$?
- $\mathbb{C}$:
  - Gödelian
  - Periods of **exp** $2\pi i$

- G.H. Hardy (1911): Orders of infinity
- If $f(x)$ is an exponential polynomial over $\mathbb{R}$ then f has only finitely many zeros in $\mathbb{R}$

Schaunel Conjecture

- $\lambda_1, \ldots, \lambda_n \in \mathbb{C}$ linearly independent over $\mathbb{Q}$
- Then **t.d.** $\mathbb{Q}(\lambda_1, \ldots, \lambda_n, e^{\lambda_1}, \ldots, e^{\lambda_n}) \geq n$
- $F(e, e^e, e^{e^e}) = 0$?

Hovanskii

- $$\begin{cases} f_1(x_1, \ldots, x_n, e^{x_1}, \ldots, e^{x_n}) = 0 \\ \vdots \\ f_n(x_1, \ldots, x_n, e^{x_1}, \ldots, e^{x_n}) = 0 \end{cases}$$
- Jacobian non-zero
- This has only finitely many solutions in $\mathbb{R}^n$
- Uniformly in parameters

- **Elimination Theory** (Wilkie 1991)
- Every formula $\phi(\overline{y})$ is equivalent to an existential formula:

$$\exists\, x_1, \ldots, x_n \, [H(\overline{x}, \overline{y}) \text{ has a solution }]$$

Where H is the Houvanskii system

- $(1+p)^* : \mathbb{Z}_p \longrightarrow \mathbb{Z}_p$
-

$$\begin{array}{cc} \mathbb{Q}_p, & \mathbf{exp} \\ \Big| & \Big| \\ \mathbb{Q}_p(\alpha), & \mathbf{exp} \end{array}$$

- $\mathbb{Q}_p$, trigonometric has existential elimination
- This is non-Gödelian